Application of American Monte Carlo to Counterparty Risk Calculations

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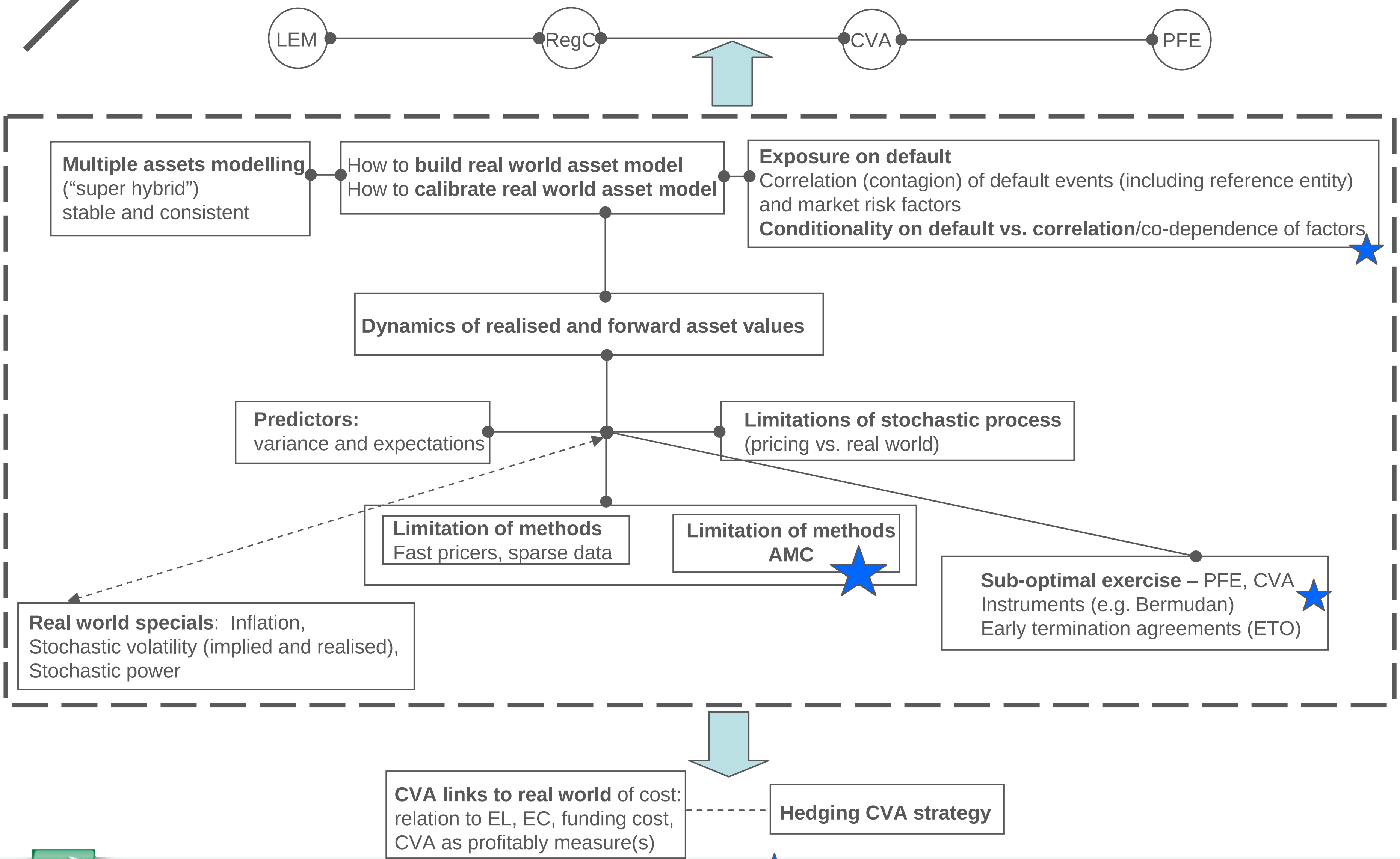
Map of risk quant - real world measure (a fragment)



Agenda

- Background
  - **American Monte Carlo**
  - **Counterparty risk**
  - **Applying AMC to risk**
- General method limitations
  - **Common issues**
  - **Parameter choices**
- Risk considerations
  - **Regressing for high percentiles**
  - **How long between coefficient updates**
  - **Counterparty conditionality**
  - **Subjectivity of optimal exercise**

Background | **American Monte Carlo**

- Also known as
  - Least-Squares Monte Carlo
  - Regression Monte Carlo
- Origins in pricing early-termination derivatives
  - **Follows developments by several authors: Carriere (1996), Tilley (1993), Longstaff and Schwartz (2001)**
  - **Method:**
    - **Define exercise times as** $e = \{e_1, e_2, \ldots, e_{N_e}\}$**, cashflows as** $c_1, c_2, \ldots, c_{N_c}$ **at times** $t_1, t_2, \ldots, t_{N_c}$
    - **The value at time** t **is given by**

$$V(t) = N(t) \sup_{e_* \in e} \left\{ \mathrm{E}\left[\sum_{s:t_s > t}^{t_s < e_*} \frac{c_s}{N(s)} \middle| F(t) \right] + \mathrm{E}\left[\frac{q_{e_*}}{N(e_*)} \middle| F(t) \right] \right\}$$

where exercise e_i **has associated cashflow** q_{e_i}

    - **Optimal exercise time** e_* **is not calculated explicitly – we use a Monte Carlo routine to calculate the optimality path-wise and iteratively**

For N_u **universes, our actual calculation is defined as**

$$\overline{V}_t = N(t) \left[\frac{1}{N_u} \sum_{u=1}^{N_u} \left[\sum_{s:t_s > t}^{t_s < e_*^u} \frac{c_s^u}{N^u(s)} \middle| F(t) \right] + \mathrm{E}\left[\frac{q_{e_*^u}^u}{N^u(e_*^u)} \middle| F(t) \right] \right]$$

- Regression Monte Carlo
  - **Estimate the optimal exercise time pathwise**
    - **Calculate the continuation value via projection onto** L^2

$$\overline{V}_t = \sum_{k=1}^{N_k} \beta_t^k f^k(\underline{x};t)$$

Then define the exercise time iteratively (backwards) as

$$e_*^u = \begin{cases} e_s & E\left[V(s) < q_{e_s}^u\right] \\ e_*^u & otherwise \end{cases}$$

Here, $f(\underline{x};t)$ **is a function of observable quantities** $\underline{x} = \{x_1, x_2, \ldots, x_{N_x}\}$

  - **Approximate using chosen form of regression**
    - **Linear least squares**
    - **Non-linear least squares**
    - **Weighted least squares**
    - **Regression splines**

  - **The linear least squares approach:**

Minimise $$\sum_{u=1}^{N_u} \left(\overline{V^u}(t) - \sum_{k=1}^{N_k} \beta_t^k f^k\left(\underline{x^u};t\right) \right)^2$$

- Benefits: simple to understand, very efficient

- Drawbacks: may not capture the shape of continuation value well

Background | **American Monte Carlo**

- Significant developments in this area:
  - Kogan, Haugh (2001) / Rogers (2002) / Andersen, Broadie (2004):
    Developed approaches for estimating upper bound prices of callables
  - Glasserman, Yu (2002)
    Provided results on the appropriate numbers of simulations to use
  - Chaudhary (2005) :
    Improved efficiency using Quasi random sequences and Brownian bridges
  - Egloff (2005) / Kohler (2006)
    Applied RMC in a more general sense (i.e. without using linear least squares)
  - Cesari, Aquilina, Charpillon, Filipovic, Lee, Manda (2010):
    Gave examples on applying the technique to counterparty exposure

Background | **Counterparty risk**

- Involves the calculation of several quantities:
  - **Potential future exposures (PFEs)**

$$PFE(\alpha;t) = \inf\{x : P(V(t) > x | \tau = t) \leq (1-\alpha)\}$$

    - **Uses "real-world" measure to generate the scenarios for $V^j(t)$**
  - **Credit value adjustments (CVAs)**

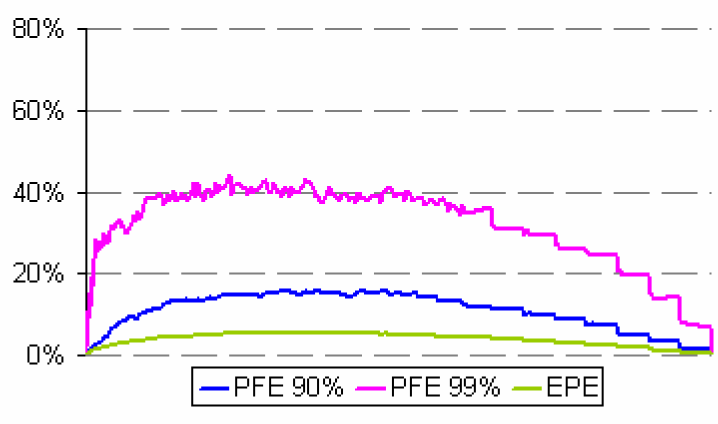
$$CVA = \int_0^T \mathrm{E}\left[D(0,t)(1-R(t))\max(V(t),0) \mid \tau = t\right] dPD(t)$$

    - **Where the expectation could be taken over the "risk-neutral" measure**
  - **Economic capital (EC)**

$$EC(\alpha) = \inf\{L : P(Loss(1yr) > L) \leq (1-\alpha)\} - \mathrm{E}[Loss]$$

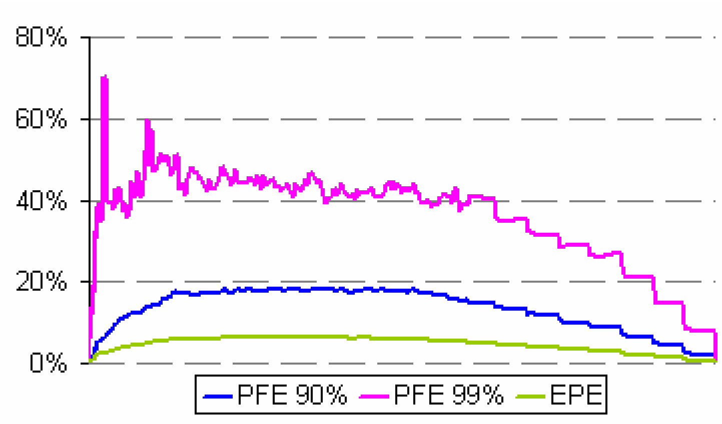
- Covers all asset classes traded
  - Equity, Commodity
  - Rates, Foreign exchange
  - Credit
- Also takes into account the credit mitigations
  - Netting
  - Collateral

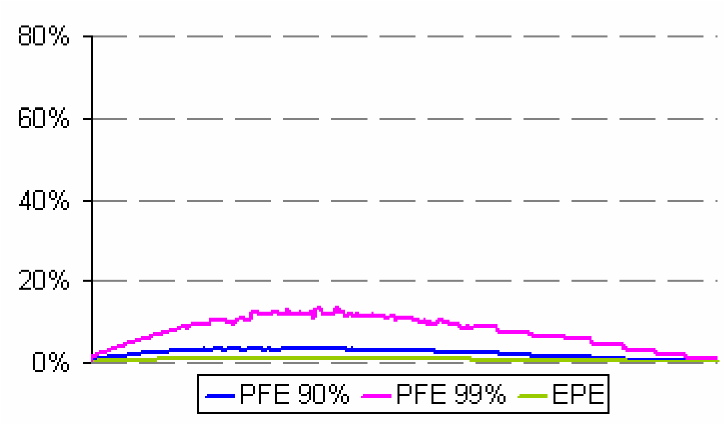
- Counterparty risk depends on the counterparty-underlying co-dependence
  - **Standard deal simulation**



With wrong-way risk …

and right-way risk





Background | **Applying AMC to Counterparty risk**

- RMC is readily applicable to counterparty risk because in doing the pricing, we get the full value paths "for free"

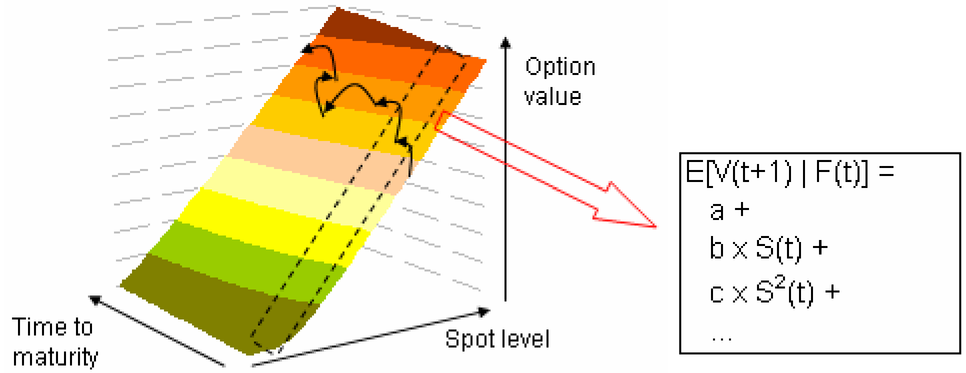
- Not just for early-termination deals – it is very useful for exposure measurement of many exotics

BUT

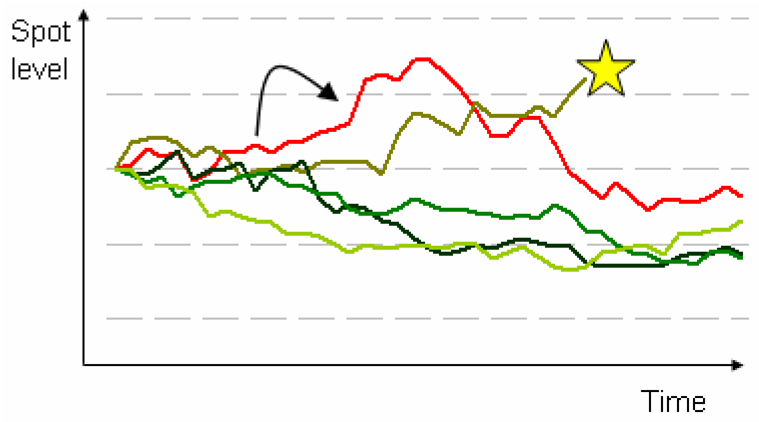
- In risk management, there are several issues to confront in addition to the pricing considerations
  - **Ensuring accuracy of prices in extreme scenarios**
  - **Calibration of "real-world" scenarios**
  - **Conditionality of counterparty default**
  - **Collateral agreements**
  - **Large-scale (covering all asset classes), consistency**

Background | **Applying AMC to Counterparty risk**

- To calculate the exposure, we need two phases:
  - **Regression Phase (A) – determine the coefficients (exercise decisions)**

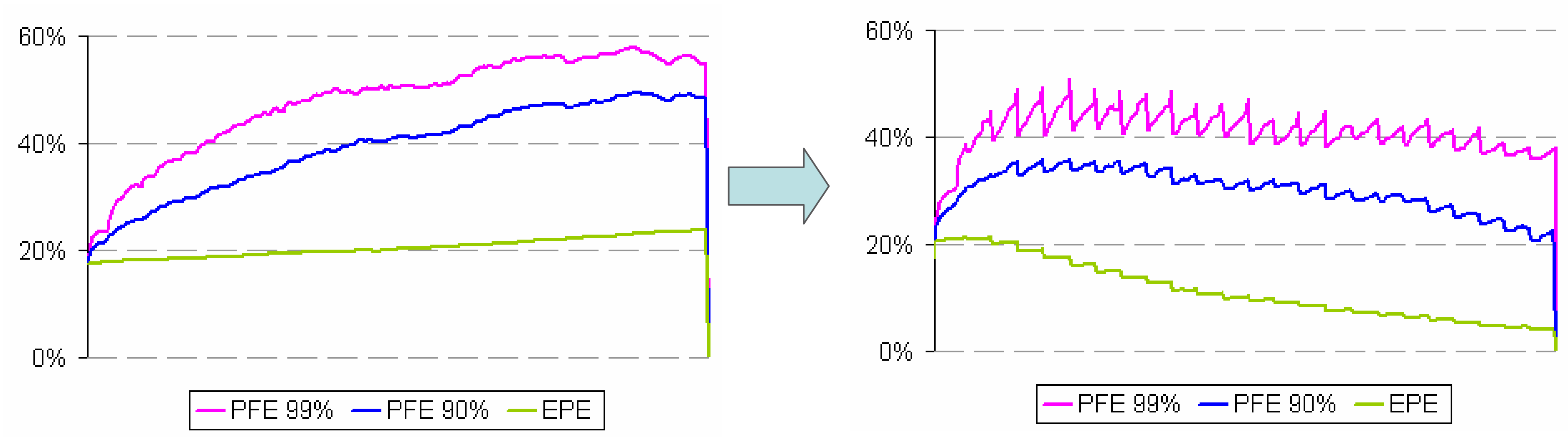


  - **Risking Phase (B) – calculate the values and termination times**



- These phases can be carried out independently of one-another

- Example: European Equity Option *to* Bermudan Equity Option



- General effects of including early-exercise:
  - Higher NPV
  - Lower values at long end
  - Peak value change *difficult to predict*
  - CVA change *difficult to predict*

Background | **Applying AMC to Counterparty risk**

	Regression Phase (A)	Risking Phase (B)
Scenarios simulated	Market-implied	Historically-implied
Direction of traversal	Backwards	Forwards
Calculation loop order	(1) Time (2) Simulation	(1) Simulation (2) Time
Includes discounting	Yes	No
Computational complexity	$O(3stk^2)$	$O(stk)$
Final output	Coefficients	Valuations
Number of simulations needed	Very large	Large

General method limitations | **Common Issues**

- Known estimation biases for early-termination deals
  - **Low bias caused by simulation error leading to suboptimal exercise**
  - **High bias caused by Jensen's inequality**
  - **Overall bias difficult to quantify**

- Choice of regression not always obvious
  - **Cash payment?**
  - **Basket options?**
  - **CDOs?**

- Technical difficulties
  - **Slow – many simulations are required for regression estimates**
  - **Possible to exhaust memory limitations – all asset prices need to be stored for all times and all paths**

General method limitations | **Regression in practice**

- Sometimes we can achieve prices which violate the bounds of the deal pricing (assuming non-negative rates)
  - **Negative values for vanilla options**
  - **Values above unity for digitals**
  - **etc.**

- How do we know if a given choice of basis functions is “good”?
  - Small in size
  - Low in variance
  - Covers the major risk factors and sensitivities
  - Produce regression estimates that are highly sensitive to the number of simulations

General method limitations | **Parameter choices**

- How to discount cashflows?
  - **We need to choose our measure and simulate all factors with the applicable drift in each case**
  - **The valuation will be the same for all measures so the risk phase is not affected**

  - **Main measures used in practice:**
    - **Spot measure** $N(t_j) = \exp\left(\int_0^{t_j} r(u)du\right)$
    - **Terminal measure** $N(t_j) = B(t_j, T)$
    - **Rolling measure** $N(t_j) = B(t_j, t_{j+1})$
    - **Annuity measure** $N(t_j) = \sum_{i=j+1}^{N} \tau_i B(t_j, t_{i+1})$

  - **Different pricing measures can give lower variance of results**
    - **e.g. Price of a zero-coupon bond**

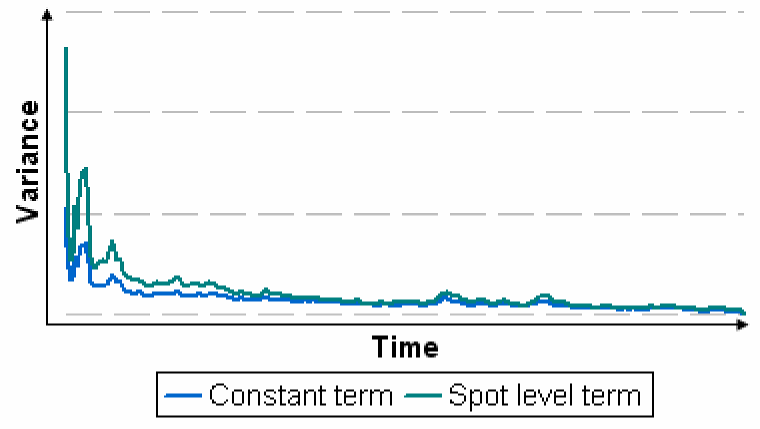
Under terminal measure:
(zero variance)

$$V(t) = B(t,T)\mathrm{E}^T\left[\frac{1}{B(T,T)}\middle|F(t)\right] = B(t,T)$$

Under spot measure:
(high variance)

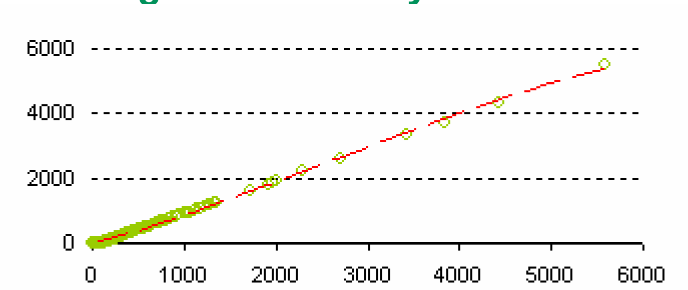
$$V(t) = M(t)\mathrm{E}^0\left[\frac{1}{M(T)}\middle|F(t)\right] = \mathrm{E}^0\left[e^{-\int_t^T r(u)du}\middle|F(t)\right]$$

- How many simulations?
  - **How do we know how good a regression has been?**
    - **R^2 is a bit meaningless … We can use bootstrap / jackknife techniques to estimate the variance of the regressions**

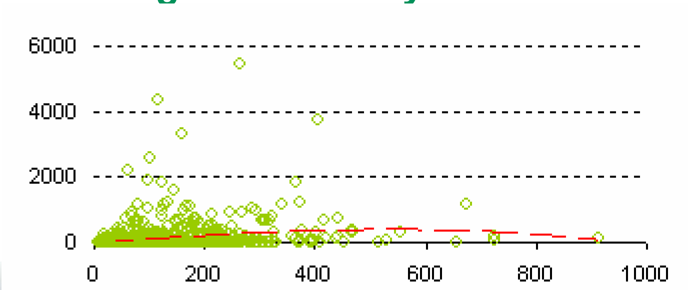


  - **How about the regression close to the initiation of the deal?**
    - **20yrs maturity, S0 = 100, K = 100**

Regression at 19yrs



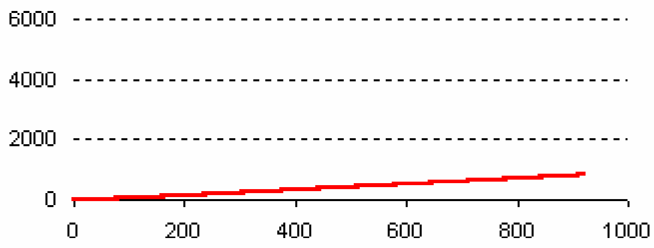
Regression at 5yrs



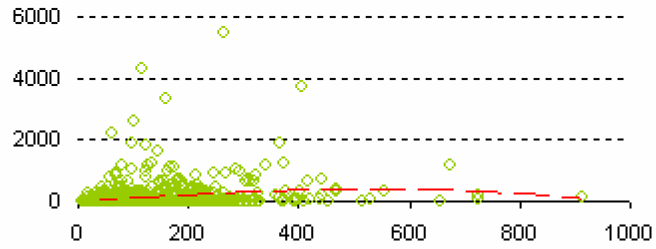
- For some deals, e.g. at-the-money digital option, the highest exposures occur where the majority of paths are generated
- In general, two problems can occur when estimating high percentiles:

1. **Insufficient density of paths in those payoff regions**

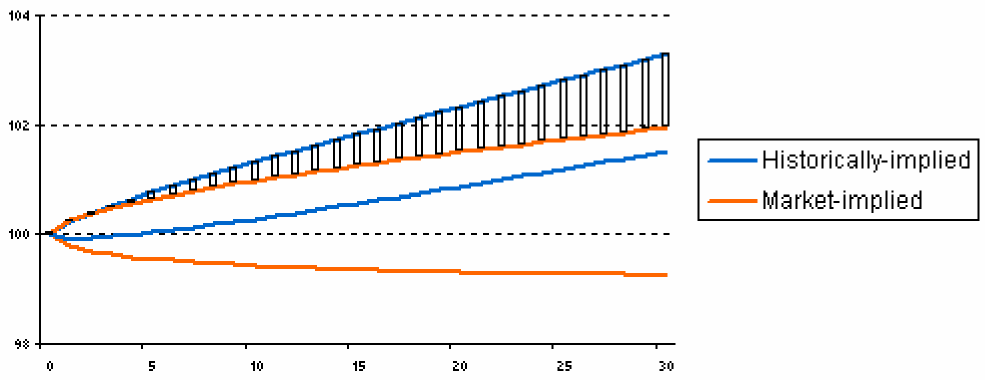
Theoretical



Simulated



2. **Scenarios used for risking go outside the regions for which the coefficients were estimated**
   - **Different drifts**
   - **Different volatilities**



Risk considerations | **Regressing for high percentiles**

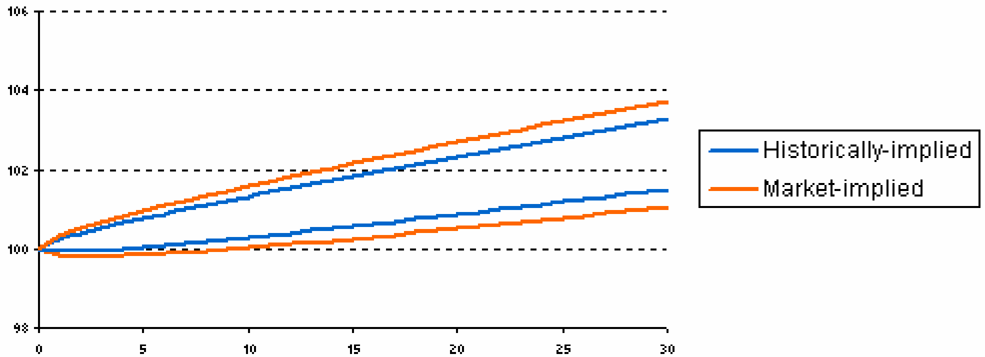
- To resolve these, a range of solutions have been proposed
  - **Importance sampling – we can use a change of measure:**

$$V(t) = M(t)\mathrm{E}^M\left[\frac{1}{M(T)}C(T)\middle|F(t)\right] = N(t)\mathrm{E}^N\left[\frac{1}{N(T)}C(T)\middle|F(t)\right]$$

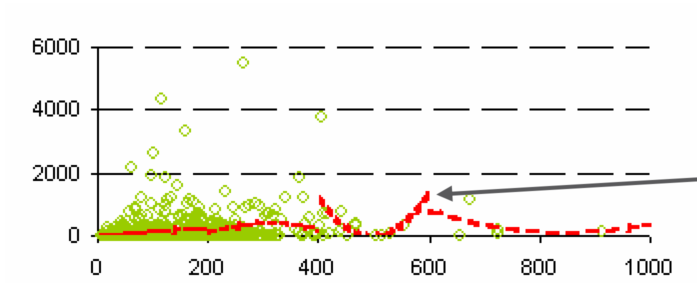
Such that under N **,** $dS_i(t) = \mu dt + \sigma_i(t)dW_i(t)$

We can choose μ **to give optimal fit to the real-world distribution and then cover all simulations**

Troublesome with multiple variables!

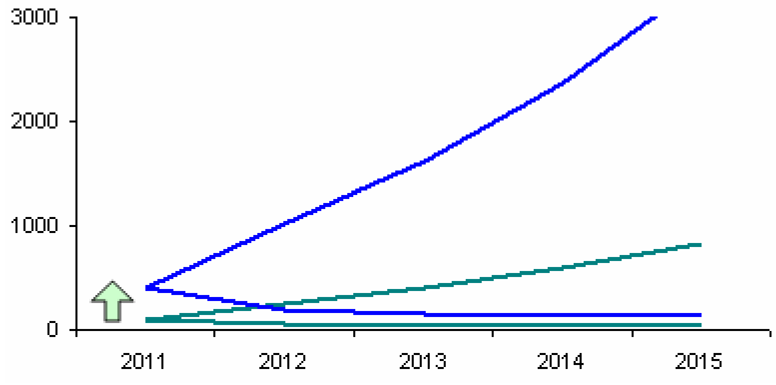


  - **Bundling – we can group the observations into buckets and perform segmented regression**

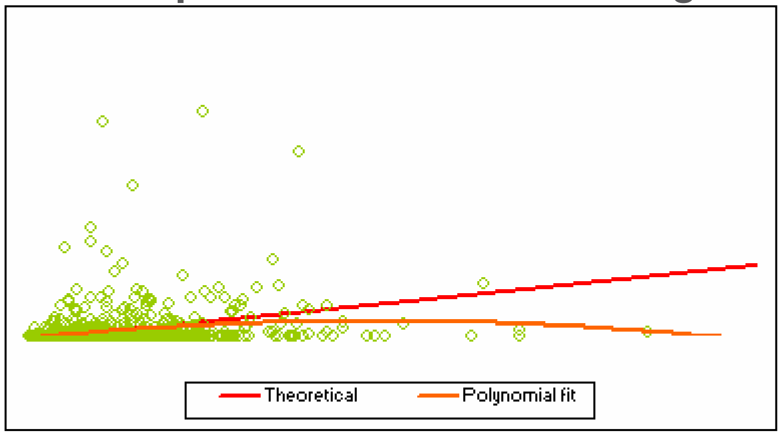


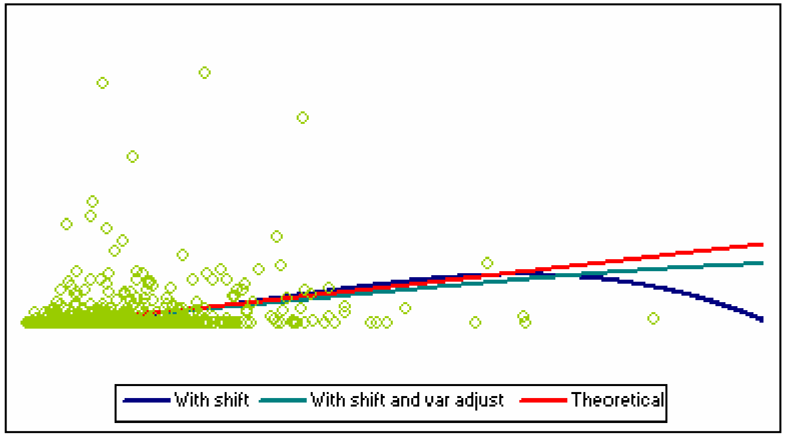
Not joined up!

- Simulation shifting
  - **A simple approach – shift the starting state of the variables**



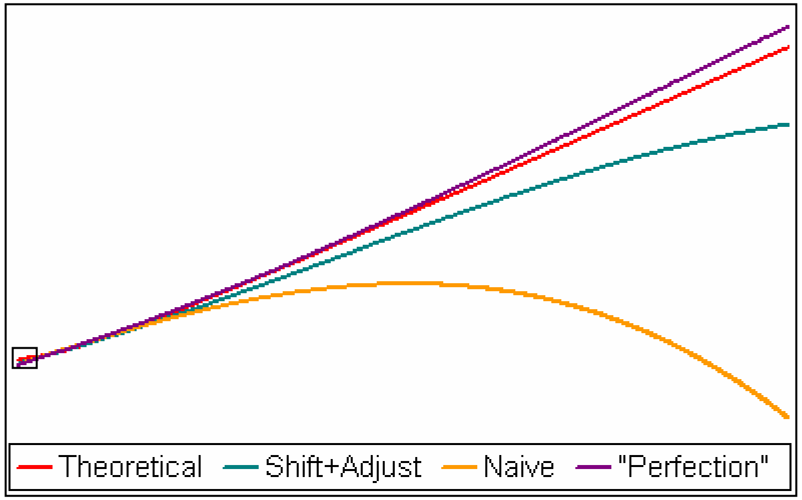
  - **Split into buckets and weight according to variance of each bucket**

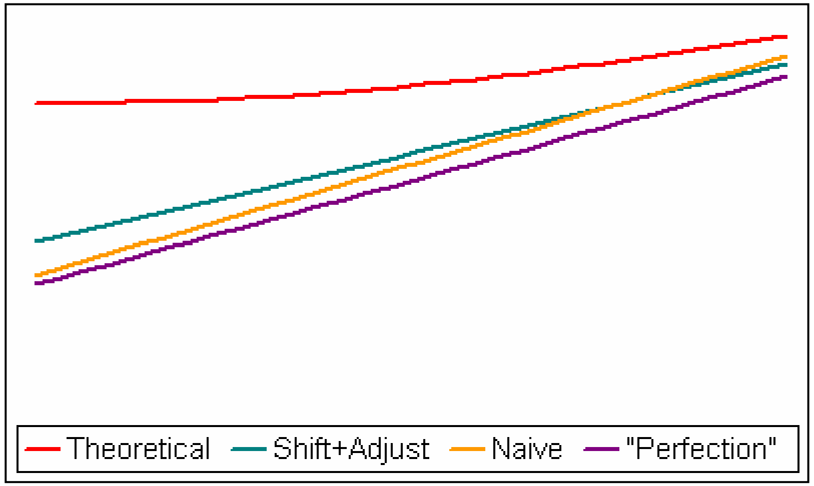




Risk considerations | **Regressing for high percentiles**

- Can we get a "perfect" fit?
  - **Suppose we are able to generate points such that distribution = constant**



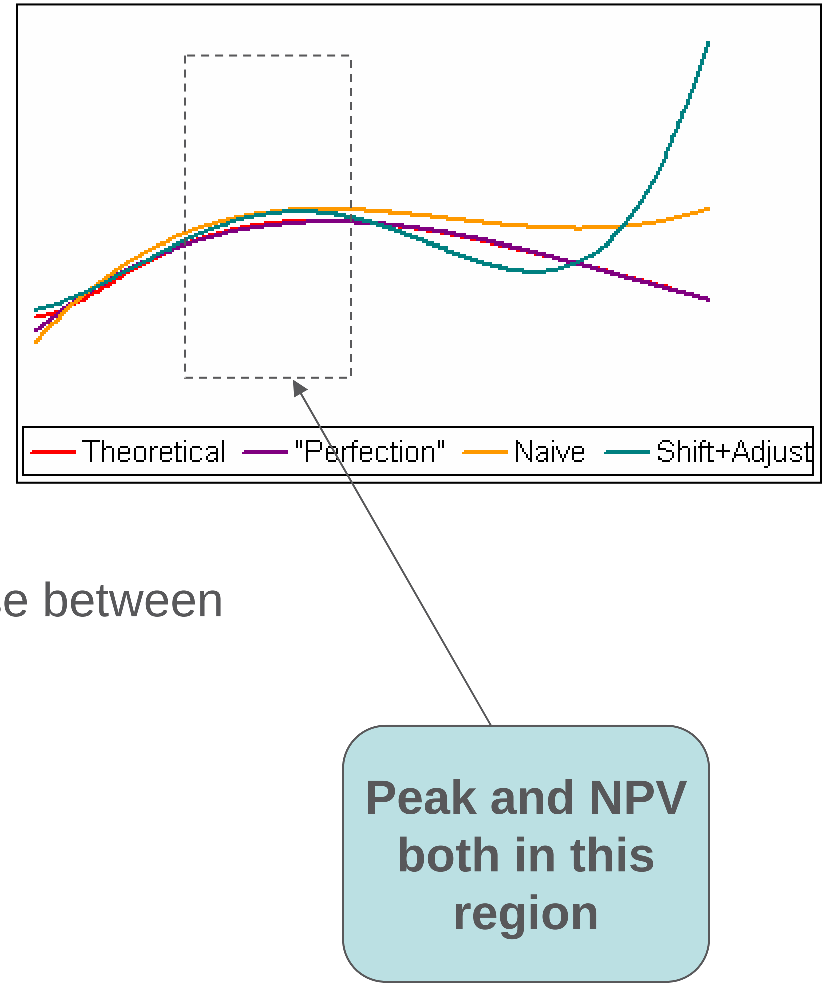


  - **… we compromise the fitting at the lower end**

resulting in a ~10% error in the price ATM

Risk considerations | **Regressing for high percentiles**

- No such concerns with up-and-out barrier options
  - **In this case,**
    peak ~ start point
  - **Our fit overestimates peaks due to discrete barrier crossing**



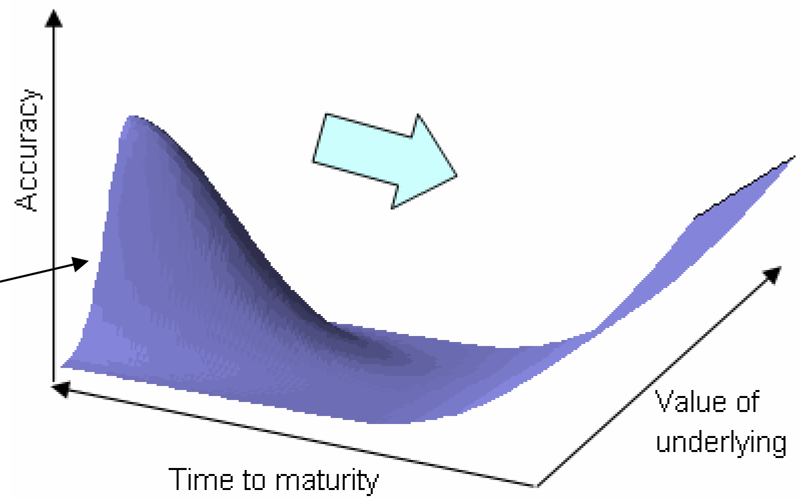
- Overall, regression is a compromise between
  - **Matching EPE -> CVA**
  - **Matching PFE peak**

- We can control this by altering
  - **Distribution of simulated paths**
  - **Choice of basis functions**
  - **How we split the observables region**

Risk considerations | **How long between coefficient updates**

- How many days between refreshing of the coefficients?
  - **We can guess that the aging effect is a function of**
    1. $t - t_0$
    2. $\sigma_{implied}\ ,\sigma_{realised}$
    3. $\#\, simulation\ (N_U)$

$$G\left(x_1,\ldots,\ x_{N_X};N_U,T-t\right)$$



i.e. how often we need to refresh $\Rightarrow N_U$

- Choose some target variance of the NPV, e.g. 5%
- Set N_U such that

$$\mathrm{E}[G] = \int \ldots \int G\left(\hat{x}_1,\ldots,\ \hat{x}_{N_X};N_U,T-t\right) d\hat{x}_1 \ldots d\hat{x}_{N_X} < 5\%$$

- Incorporating the counterparty-payoff co-dependence
  - **Estimate $F(V(t)|\tau = t)$ in the cases where the processes generating V and τ are not independent**
  - **The slow approach – brute force**
    1. **Simulate $\{x_i\}$ and τ for a "very large" number of simulations**
    2. **Retain only those paths where $\tau < T$, the longest deal maturity**
    3. **Value each deal and aggregate deal population to get $F(V(t)|\tau = t)$**
  - **The fast approach – moment matching (Buckley, Wilkens, Chorniy (2011))**
    1. **Estimate μ_{CP} and σ_{CP} from the counterparty correlation and rating parameters**
    2. **Simulate $\{U_i\}$ for a "large" number of simulations**
    3. **Shift simulations by μ_{CP} and scale by σ_{CP} to get conditional returns distribution**
    4. **Value each deal and aggregate deal population to get $F(V(t)|\tau = t)$**
- This approach is entirely compatible with the AMC
- For CVA only: we can show that $\mathbb{P}(D)\mathbb{E}(Y|D) = \mathbb{E}(\mathbb{P}(D|Y)Y)$
  - i.e. we estimate the distribution of counterparty asset returns conditional on the exposure

Risk considerations | **Subjectivity of Optimal Exercise**

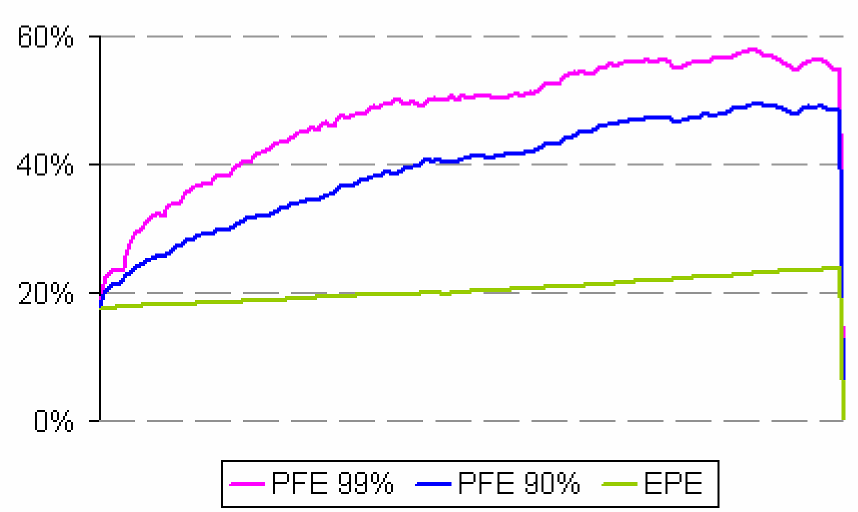
- The estimation of optimal exercise is model-dependent
  - Model differences: Number of factors; Choice of smile and skew representation
  - Authors commenting on this: Longstaff, Clara, Schwartz (2001); Andersen, Andreasen (2001)
- Some counterparties may also choose not to exercise when optimal
  - **Sophistication, infrastructure, illiquidity, …**

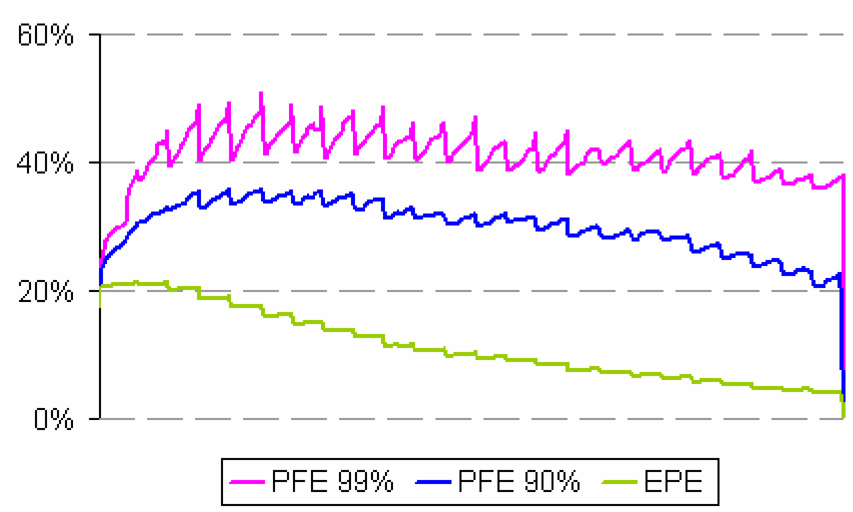
- Three questions to ask:
  1. **Does it happen?**
  2. **Is it reflected in pricing?**
  3. **Is it reflected in hedging?**

- The risk management is affected by all three (1st question is based on real world vs. risk neutral world difference).
  - If answer "yes/no/no" - risk management may choose to use real world modelling affecting the methods

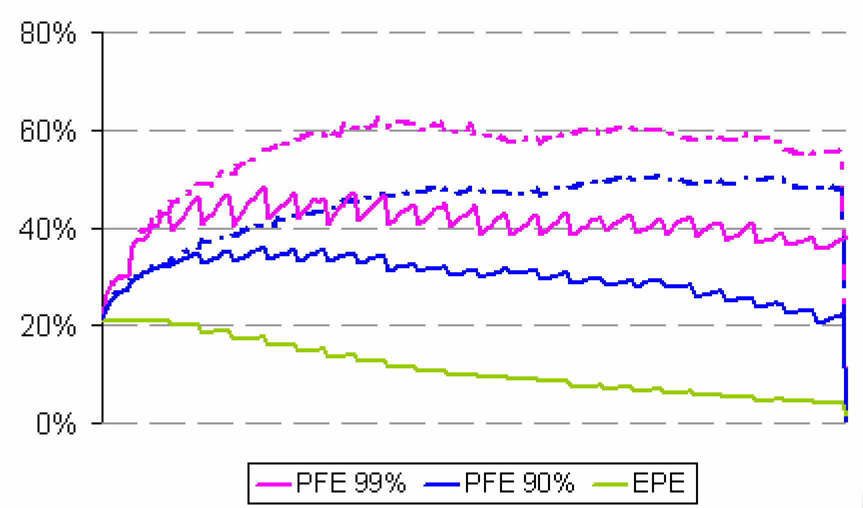
Risk considerations | **Subjectivity of Optimal Exercise**

- Example: European Equity Option *to* Bermudan Equity Option





- Allowing for optimality “slippage”:



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