Risk-Neutral vs. Real World

Application to Risk, Capital and CVA

Global Derivatives Trading & Risk Management
Paris, April 2011

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Agenda

- **Overview**
- Variance Predictors : Implied and Historical data
- Expectation Predictors : Forward Prices and Alternatives
- Limitations of Stochastic Processes : A Few Interesting Examples…
- Dynamics of Realised and Forward Asset Values
- Some Final Remarks!

Why?

The world of risk and pricing are coming closer together

- Old ways:
  - Reserves (not hedged), possibly through the cycle – historic approach
  - Pricing and hedging of derivatives – risk neutral approach

- Now almost a continuous spectrum:
  - Price – CVA – VAR (including CVA VAR Basel 3) and PFE – EC – Reg Cap
  - Also CVA brings risk and pricing views together (CVA as price of risk)

- Lack of clarity on applicability of approach for each task
- Re-inventing bicycles? Or repeating old mistakes?

2010's: *"With CVA as price of risk the counterparty PFE limits should be abolished, all we need is the right price for the risk"* – pricing quant
Late 1990's: *"With EL, EC and RAROC framework the counterparty PFE limits should be abolished, all we need is the right charge for the risk"* – risk quant
Spot the difference…

"Market is the best predictor…", perhaps yes, but of ***what*?**

What?

In this presentation:

- Variance predictor: historic volatility vs. implied
  - Case study: Equity, FX
- Expectation predictor: role of market forwards and alternatives
  - Case study: FX, Equity volatility
- Limitations of stochastic processes
  - Drifts, percentiles, jumps
- Dynamics of realised vs. forward asset values
  - Case study: Inflation, Equity volatility, FX, IR

Not in this presentation, but other important aspects of risking vs. pricing:

- Discussion of methods – applicability and limitation in pricing vs. risking world (e.g. use of AMC for either and its limitations)
- Discussion of structural/economics approaches (e.g. calibration to extreme events and regime changes such as long term prediction of inflation at high percentiles)

(watch this space!)

The aim of this presentation is to outline some of the problems and illustrate them with examples. However we will not be proposing all the solutions!

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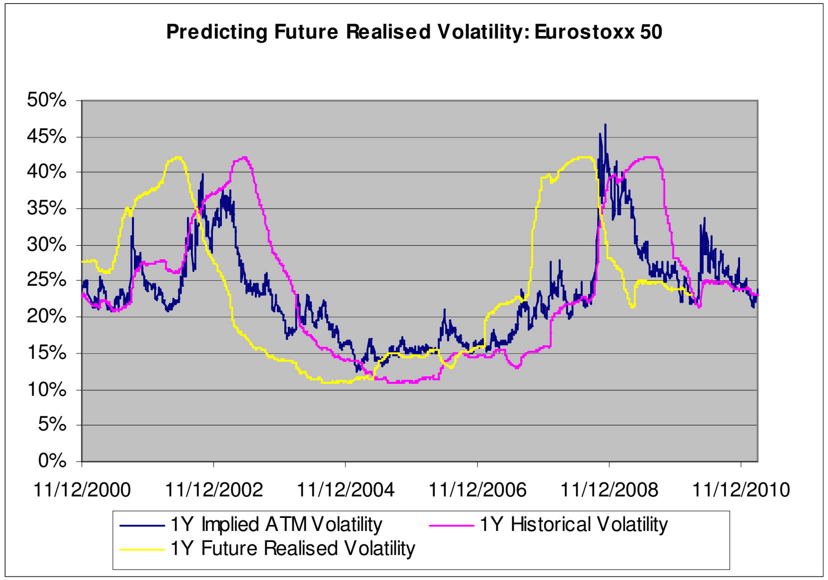
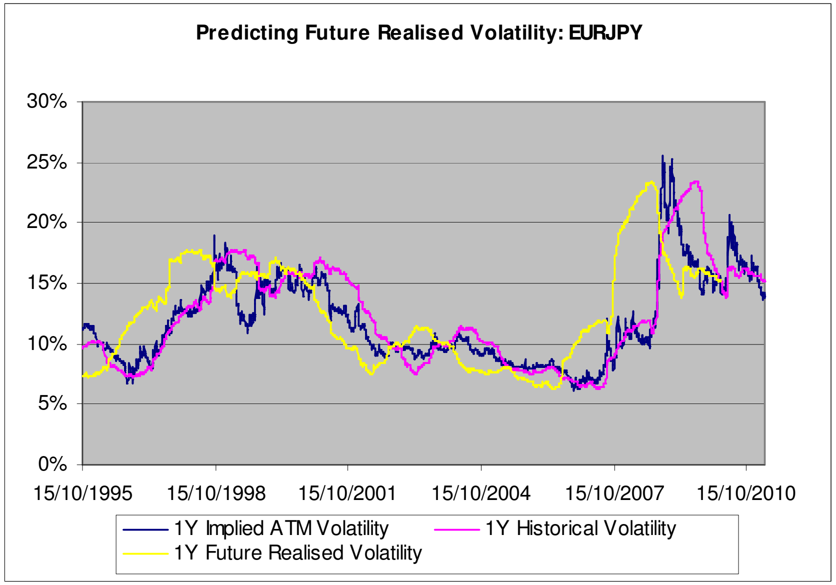
Variance Predictors : Implied vs. Historical

- Estimator for the future realised volatility of the spot





- Compared to the historical volatility, the implied volatility is more responsive to market moves
- In the periods after crises implied volatility decreases sooner than the historical volatility
- Neither can predict an upcoming crisis!

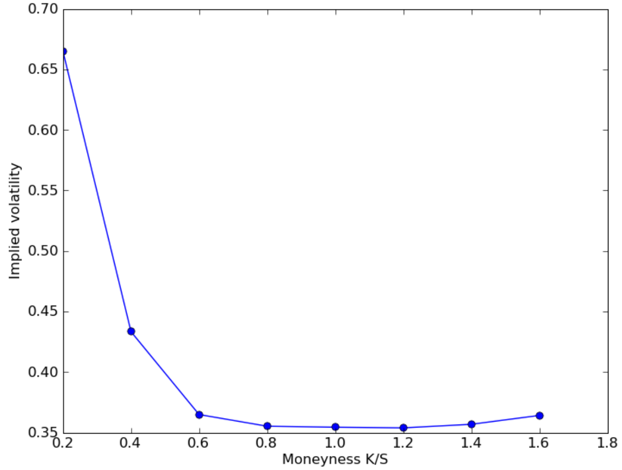
Variance Predictors : Implied vs. Historical

- The prediction for longer maturities is poor by both volatilities: implied and historical
- Regression analysis indicates implied volatility performs marginally better as an estimator on average, however it tends to “overshoot” and is less stable
- Practical issues for implementation in a risk context:
  - very long tenor predictions (up to 50 years...)
  - consistency of volatility and correlation
  - data availability and effects of liquidity
  - meaningfulness of strike dimension... (see next slide)

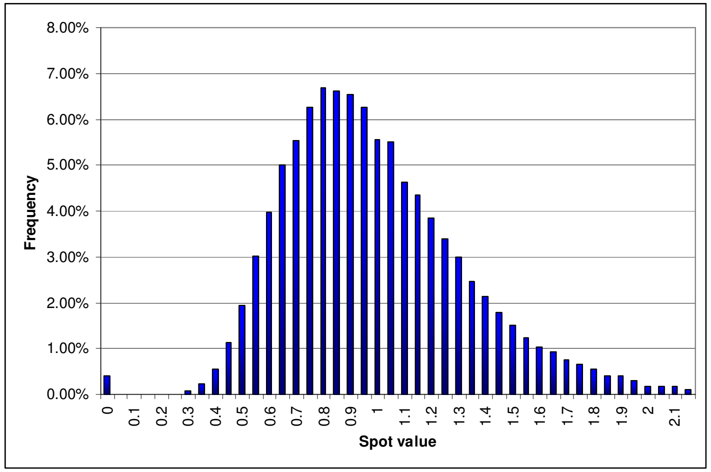
Variance Predictors : Predictive Power of Skew

Market quotes for vanilla options exhibit implied volatility skew

- Postulating the "true" process of the stock is a geometric Brownian motion with constant volatility (35%) and jumps to default (zero value with probability 0.4%/year) the following results are obtained:



Implied volatility as a function of moneyness for vanilla options



Spot probability density function

- Implied volatility at a given strike is a bad predictor.

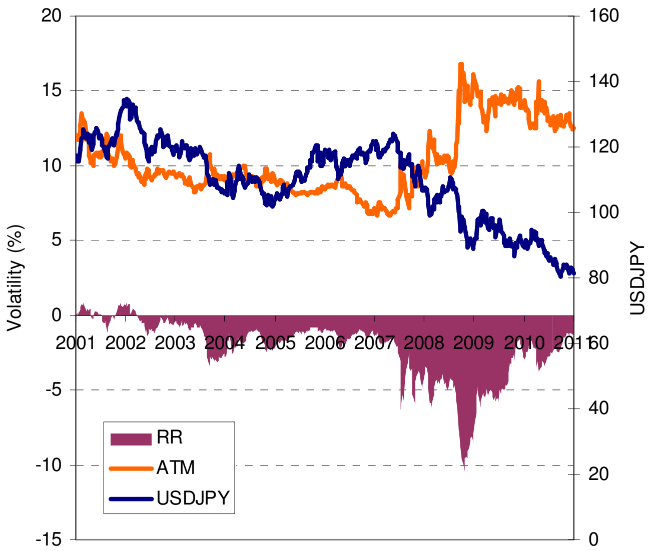
The example above can be reproduced using local volatility model (though calibration at very low strikes will be difficult).The skew is equivalent but the stock behaviour is radically different:

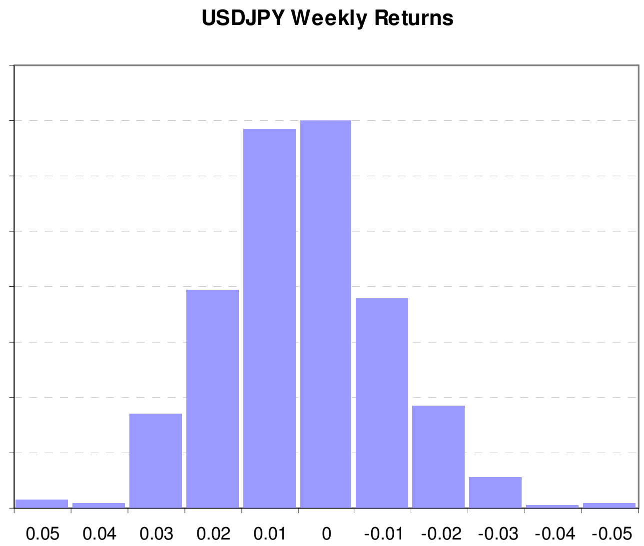
- Local volatility model suggests the spot volatility increases when the spot decreases while for constant volatility the skew is due to jumps to default.
- We did not reflect the postulated model but does it reflect real world? Equity maybe, what about FX?

Variance Predictors : Predictive Power of Skew

Which behaviour is the most realistic?

- The increase in realised volatility may be true for equity but for FX, analysis shows the opposite
- The price of a risk reversal (RR) tells us about the value of calls to puts: ($RR_{25}=\sigma_{call,25} - \sigma_{put,\ 25}$)
- The mostly negative RR price indicates the market nearly always prices puts more highly i.e. the market predicts volatility will increase when yen strengthens.

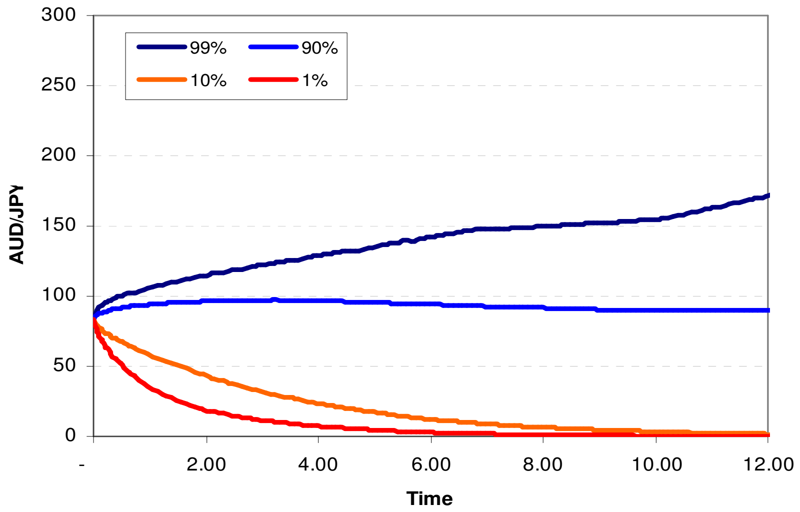




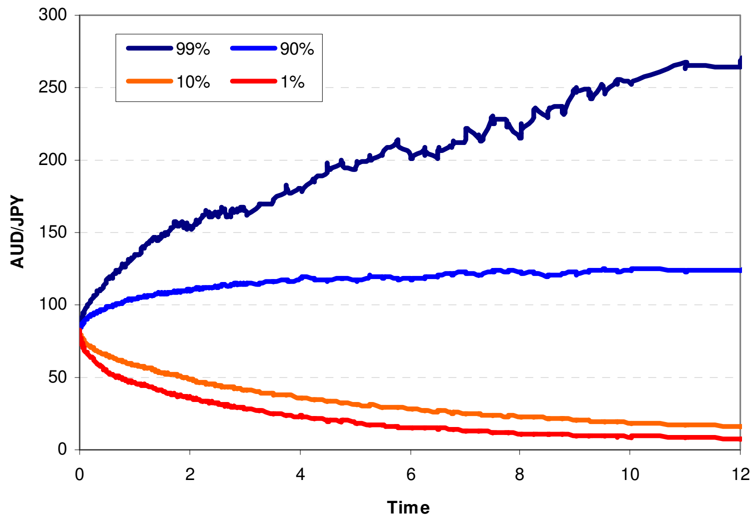
- The historical FX returns are fairly symmetrically distributed about zero so, the probability of an up or a down move are fairly even.

Variance Predictors : Predictive Power of Skew

- Using local volatility model to diffuse the FX spot may lead to “unrealistic” scenarios such as JPY/AUD parity.



FX Spot diffusion using risk neutral calibration, going to parity for 1/10 percentiles!



FX Spot diffusion using real world calibration

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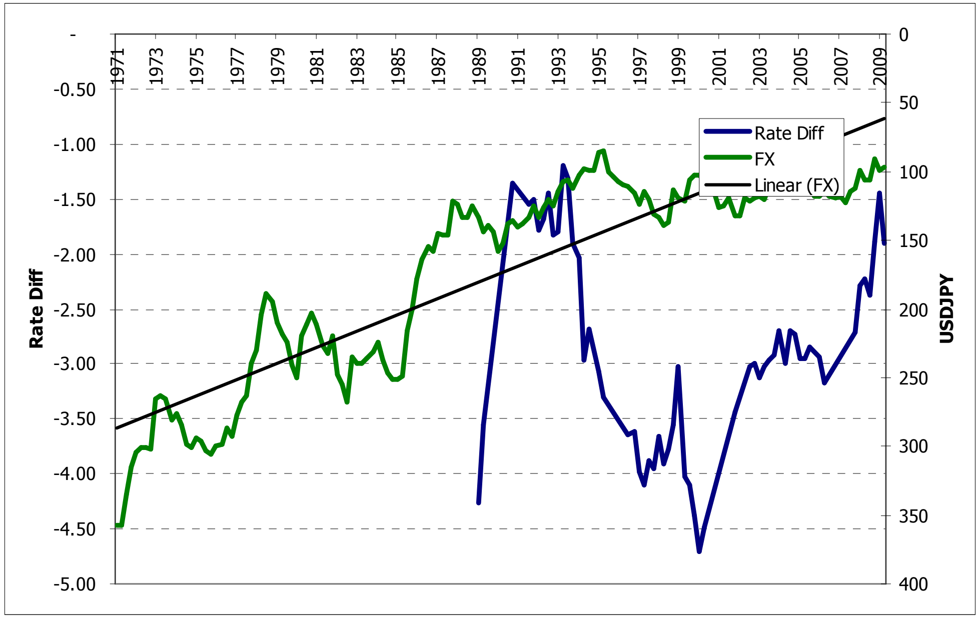
Expectation Predictors : Forward vs. Spot FX

- The FX forward, at time t, in the risk neutral world is the expected value of the future spot FX such that there is no arbitrage between the two currencies.
- In the real world arguments such as purchasing-power-parity also suggest that the forward may be a good expectation (at least over the long term)...



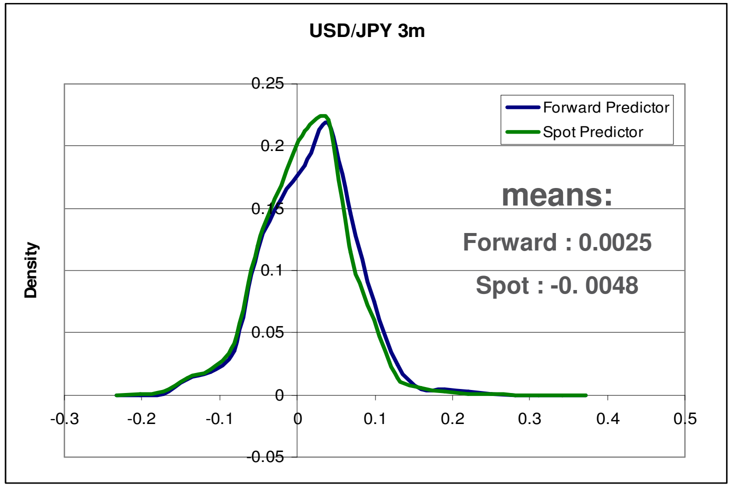
→ Seems generally true for long term predictions

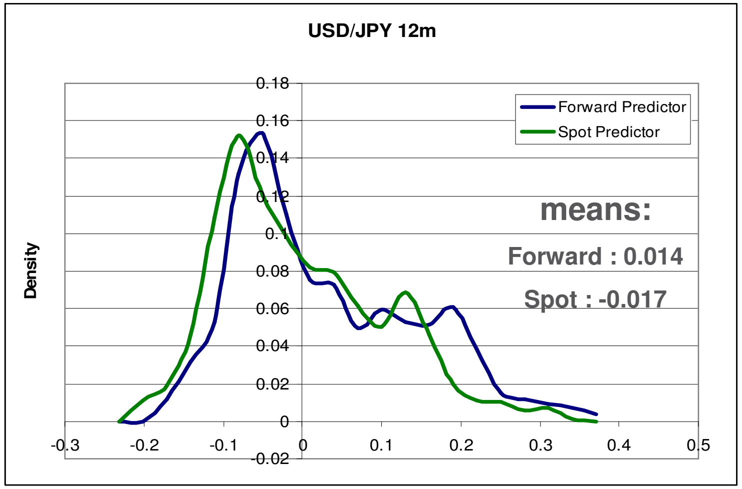
Expectation Predictors : Forward vs. Spot FX

- What about shorter term predictions...
- Simple test - look at the density plots of the normalised returns

$$\frac{F(t)_0 - S_t}{S_t}$$

using the F = the forward price or the spot price and S_t = the spot price realised at time t later for non-overlapping periods.





- If the extra information in the forward price (the interest rate differential) is relevant we should see less variance in the estimator...
- ...or we could see additional variance from the added stochasticity

- Little difference observed between the distributions, variance identical.
- Inconclusive...

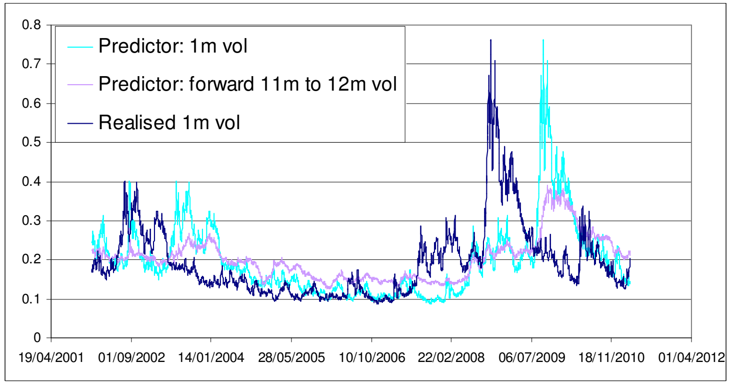
Expectation Predictors : Future Implied Volatility

Two possibilities to use term structure as of COB date:

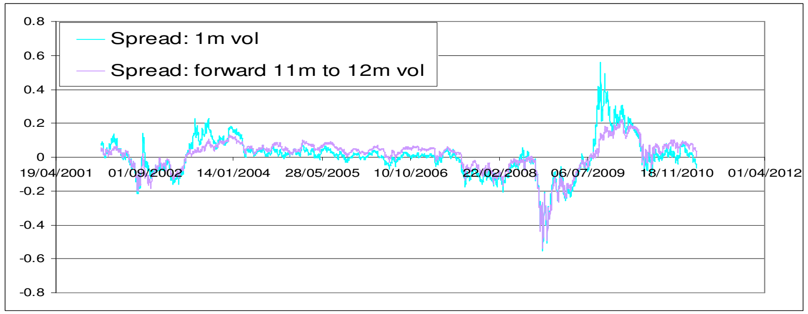
- "Curve Pushing" → Term structure is a structural effect that persists through time
- Forward Volatility → Forward volatility from today's curve contains information about expected future volatility:

$$\Sigma^2(t_0;T_1,T_2) = \frac{T_2\Sigma^2(t_0;0,T_2) - T_1\Sigma^2(t_0;0,T_1)}{T_2 - T_1}$$

Data analysis for SPX ATM volatility



$$\Phi = \Sigma_{estimated} - \Sigma(t_0 + 11m;0,1m)$$



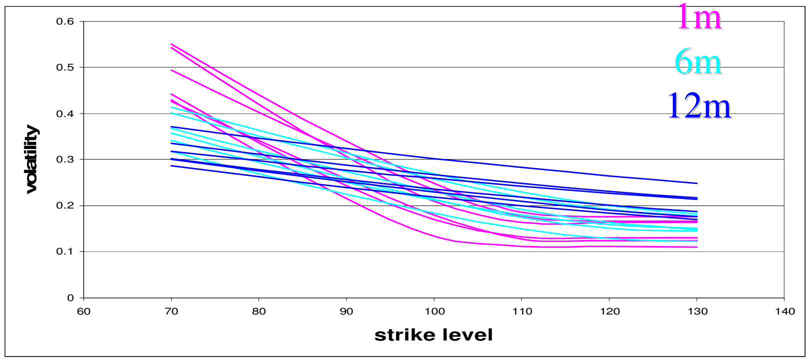
	curve pushing	forward vol
mean	-0.5%	-1.0%
1%-ile	-42.2%	-41.7%
10%-ile	-20.2%	-18.6%
90%-ile	23.0%	16.0%
99%-ile	41.3%	20.1%

→ Forward volatility seems to perform better overall (at least it seems to correctly predict that volatility will come down after a crisis!)

Expectation Predictors : Future Volatility Smile/Skew

- Similarly to ATM case could use spot or forward volatility skew
- Forward volatilities work for ATM case, but skew is not preserved with this approach (forward skew is generally *flattened*)…
- … but we know that generally skew *shape* persists through time:

• e.g. SPX implied volatilities over a selection of dates (2m intervals)
•Shape persists, level moves with ATM vol



Analysis of spread Φ for forward predictors 3m versus 6m to 9m, March 2007 – March 2011:

	curve pushing							forward vol						
strike	70	80	90	100	110	120	130	70	80	90	100	110	120	130
mean	-1.1%	-0.6%	-0.5%	-0.5%	-0.5%	-0.4%	-0.4%	-10.9%	-7.3%	-3.7%	-0.5%	1.8%	1.9%	1.0%
1%-ile	-37.1%	-35.0%	-33.8%	-32.4%	-30.9%	-27.9%	-25.0%	-43.0%	-39.2%	-35.1%	-31.1%	-27.8%	-25.3%	-24.0%
10%-ile	-20.7%	-19.1%	-17.7%	-17.4%	-16.6%	-15.1%	-13.5%	-26.9%	-23.3%	-19.8%	-16.3%	-13.7%	-12.3%	-11.8%
90%-ile	14.0%	14.2%	14.1%	14.0%	13.3%	12.2%	11.1%	0.7%	5.3%	9.0%	11.9%	13.7%	13.6%	13.0%
99%-ile	25.1%	25.2%	24.9%	24.2%	23.0%	21.0%	19.2%	3.4%	7.6%	11.1%	14.3%	16.4%	17.0%	16.3%

→ Curve pushing seems to do better; forward volatility tends to underestimate realised implied volatility, especially for low strikes where skew is pronounced

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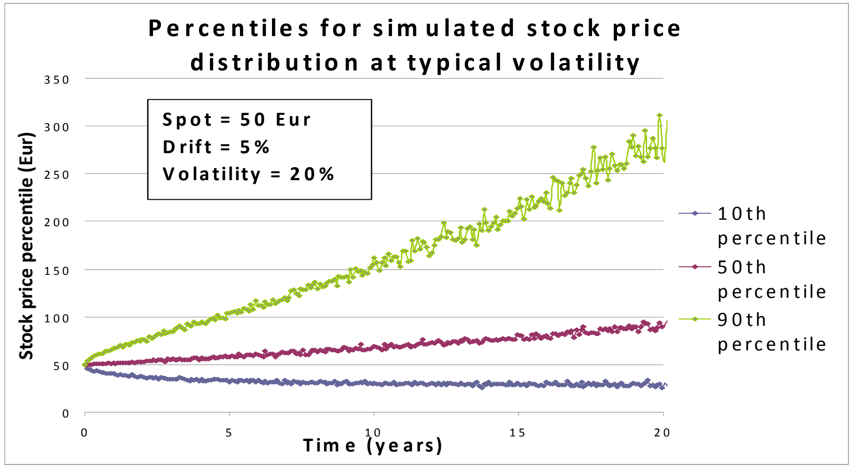
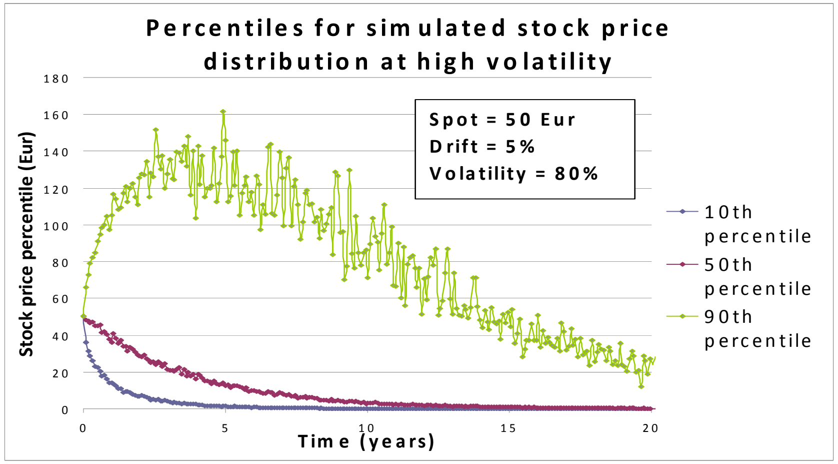
Limitations of SPs : Lognormal Equities

Stock prices often modelled as lognormal

$$S_t = S_0 \exp\left[\left(\mu(t) - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}\varepsilon\right]$$



- High volatilities can make the drift term negative
  - Common in a risk-neutral context
  - Problematic in a real-world (risk) context

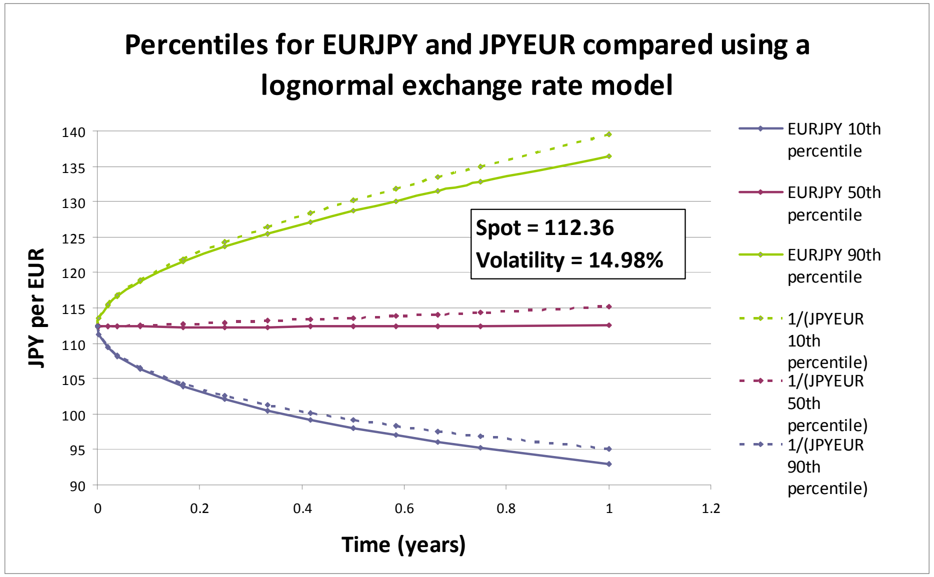


- Consider the risk of a long maturity call option on the stock
  - We show **no risk** at a given percentile beyond a certain future time

Limitations of SPs : Lognormal FX rates

- Exchange rates can also be modelled by a lognormal process

$$S_t = S_0 \exp\left[\left(r(t) - r_f(t) - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}\varepsilon\right]$$



- In a risk-neutral context we can model S or 1/S and as long as we remember the change of numeraire adjustment, the expectations will be in agreement

- However given points on the distribution will not agree, e.g. the percentiles...
- ... but the prediction in the real-world (the risk) should be unique!
- We can adjust the drift so that the risk agrees but then the expectation will be different
- Simple consequence of the asymmetry of the distribution

Limitations of SPs : Jump Processes in Risk

Jump processes have desirable features for modelling market factors in a risk context...

- **Fatter tails** – most market factors exhibit leptokurticity.
- **Intuitive motivation** – jumps correspond to the impact of significant events on market factors; whereas, a diffusion is day-to-day noise.
- **Coincidence of jumps** – “easy” to have systemic (or sectorial) and idiosyncratic jumps allowing simple “correlation” of tail events.
- **Propagation between markets** – “easy” to model the effect that shocks in one market might have in neighbouring markets (e.g., for power markets).
- **Calibration** – Not easy because tail effects are always tricky to calibrate, but one can err on the side of being slightly conservative (from a risk management perspective).
- **Percentile Risk Measures** – jumps can be awkward when estimating a given confidence interval.

Limitations of SPs : Jump Processes in Valuation

Jumps not so popular in a valuation context...

- Hedging options under jump-diffusion processes is not complete. Consider Merton's jump-diffusion model with a spot price *S* satisfying

$$dS = \mu S dt + \sigma dW_t + (J_t - 1) dN_t$$

- For a portfolio Π of an option of value *V* and a quantity Δ of the spot, in general, one cannot hedge both stochastic terms:

$$d\Pi = \left(\frac{\partial V}{\partial t} + \mu S \frac{\partial V}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} - \Delta \mu S \right) dt$$
$$+ \left(\frac{\partial V}{\partial S} - \Delta \right) \sigma S \, dW_t$$
$$+ \left(V(S^+) - V(S^-) - \Delta S^+ + \Delta S^- \right) dN_t$$

- One must make further assumptions in order to define a risk-neutral measure. Merton assumed that the jump risk was diversifiable.

Merton, R.C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3

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Dynamics of realised vs forward : Inflation

Model

- Variant* of Jarrow-Yildrim (multi-factor model based on the FX analogy: nominal rate as domestic currency, real rate as foreign and inflation index as the spot FX rate)
- No special provision for realised rate dynamics

Model summary

- The inflation index is simulated with a log-normal process.
- The drift of the index process is stochastic and driven by the simulated inflation rate. The drift is effectively the difference between the nominal and real short-rate:
- A quantity *R* is defined as the ratio between the nominal and real ZC bond prices
- *R* is simulated with a 3-factor HJM model and used in the inflation index process

$$\frac{dI(t)}{I(t)} = (r^n(t) - r^r(t))dt + \sigma_I dw_t^I$$

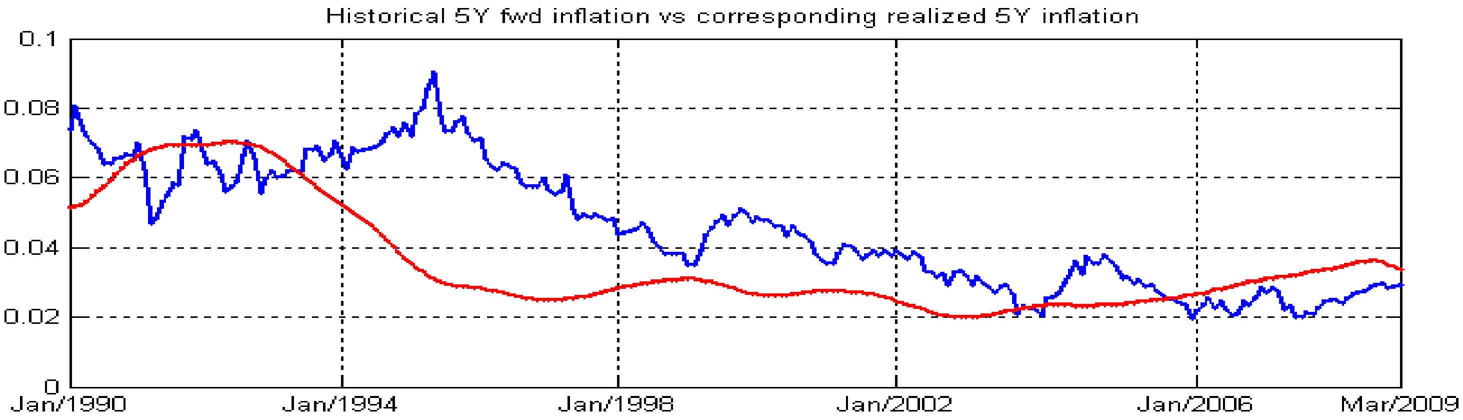
$$R(t,T) = \frac{P^n(t,T)}{P^r(t,T)} = (1 + ZC(t,T))^{-(T-t)}$$

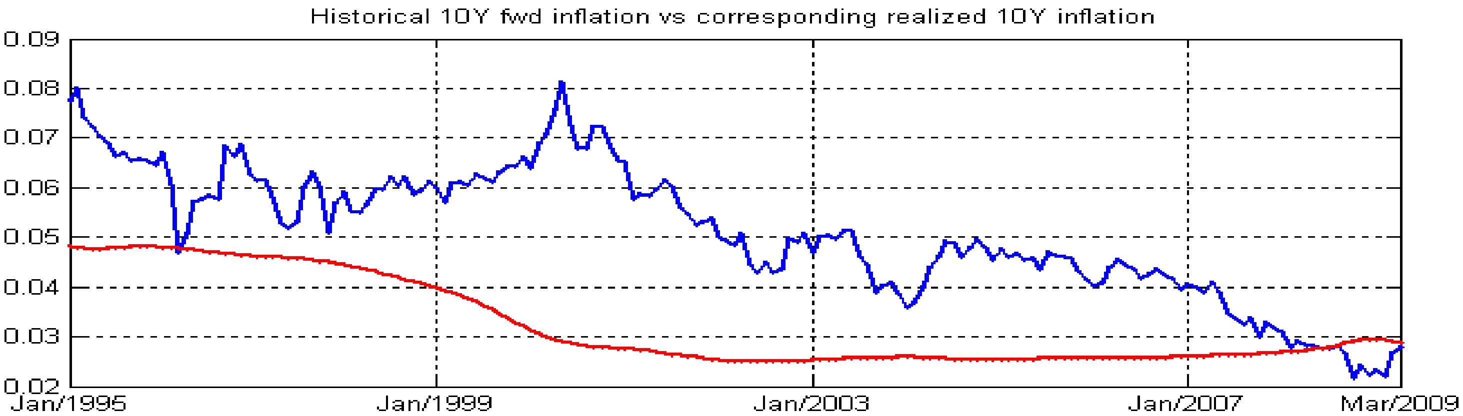
$$\frac{dR(t,T)}{R(t,T)} = \mu(t,T)dt + \sum_{i=1}^{3} \sigma_i^R(t,T)dw_{i,t}^R$$

**V. Kotecha, V. Chorniy,* Challenges In Modelling Inflation For Counterparty Risk, *Quant Congress Europe 2010*

Dynamics of realised vs forward : Inflation

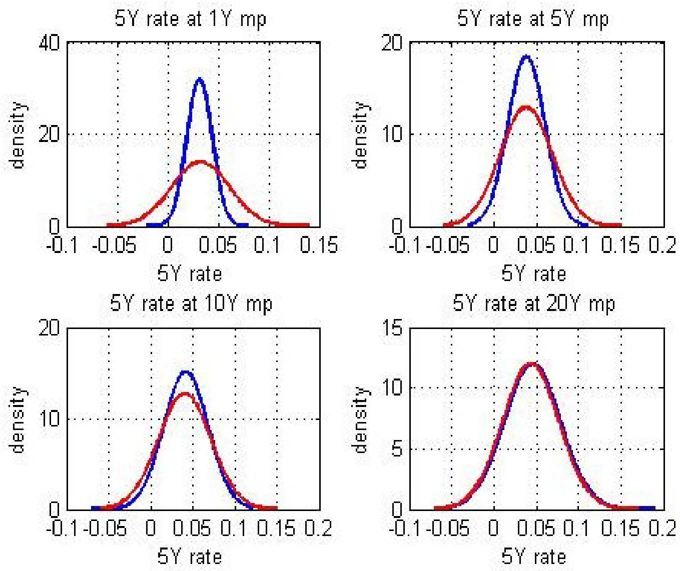
Evidence of a market "premium" - different dynamics with higher range (forward inflation in blue, realised inflation in red), 5Y and 10Y shown:



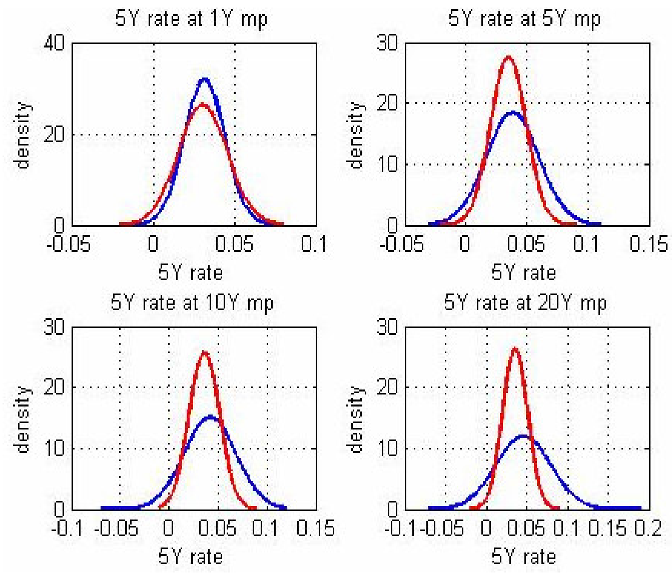


Dynamics of realised vs forward : Inflation

- Simulation of forward rate and realised rate would be expected to reflect this premium. If not - business and risk impacted
- Why? - strong impact on inflation trades: all types of compounding trades can be overcharged via PFE, Capital, possibly CVA



With full stochastic drift

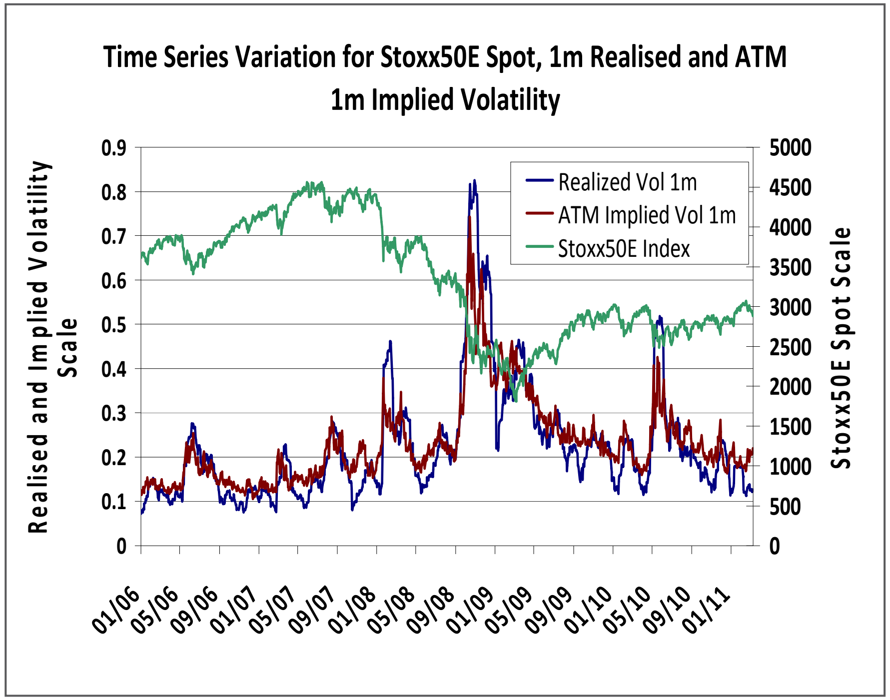


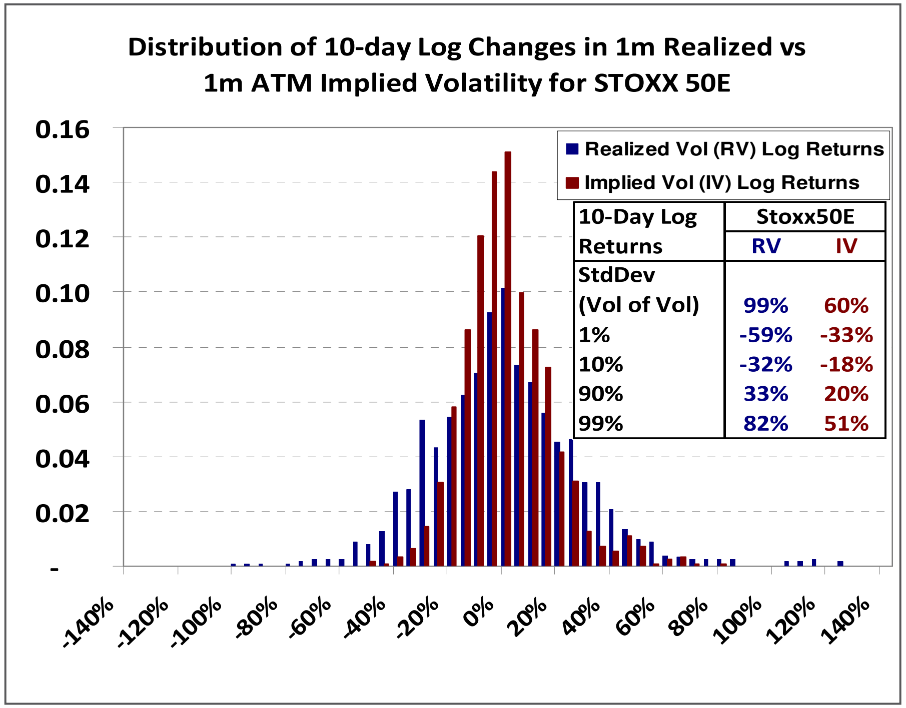
With suppression of stochastic drift

Simulated envelopes of realised inflation (red) should be narrower - only partially achieved by the suppression of stochastic drift. Single process (“pricing” approach) can reflect real world only with strong limitations.

Dynamics of Realised vs Forward : Volatility

- More extreme downward and upward moves in the short term **realised volatility** (RV) versus **implied volatility** (IV) – mainly driven by spot jumps
- Short term RV is much more volatile than ATM 1m IV
- Dynamics of short term implied volatility will not capture the tail moves in realised volatility





Dynamics of Realised vs Forward : Volatility

Example : Risk Management of Variance Swaps

- Variance Swap **payoff** for a swap starting at t_0 and maturing at T

$$\Pi_T = N\left(\frac{1}{T-t_0}\sum_{i=1}^{M}\left(\frac{S_i - S_{i-1}}{S_{i-1}}\right)^2 - K_{VaR}\right) \approx N\left(\frac{1}{T-t_0}\int_{t_0}^{T}\sigma^2(t)\,dt - K_{VaR}\right)$$

- The variance swap **strike** is set so that the swap value at time t_0 is 0 (fair price)

$$K_{t_0} = E\left[\frac{1}{T-t_0}\int_{t_0}^{T}\sigma^2(s)\,ds \,\middle|\, F_{t_0}\right]$$

- **PV move** between times t and $t+\Delta t$, assuming zero interest rates and unit notional

$$VS_{t+\Delta t}(t_0,T) - VS_t(t_0,T) = \frac{\Delta t}{T-t_0}\frac{\int_{t_0}^{t_1}\sigma^2(t)\,dt}{\Delta t} + E\left[\frac{1}{T-t_0}\int_{t+\Delta t}^{T}\sigma^2(t)\,dt \,\middle|\, F_{t+\Delta t}\right] - E\left[\frac{1}{T-t}\int_{t}^{T}\sigma^2(t)\,dt \,\middle|\, F_t\right]$$

$$\Delta VS_t(t_0,T) = \frac{\Delta t}{T-t_0}RV^2(t,t+\Delta t) + \left(1-\frac{\Delta t}{T-t_0}\right)\left(K_{t+\Delta t} - K_t\right) - \frac{\Delta t}{T-t_0}K_t$$

- Changes in variance swap value are driven by **realised variance** as well as changes in expected future **variance** (VS price)**.**

Dynamics of Realised vs Forward : Volatility

Example : Risk Management of Variance Swaps (cont'd)

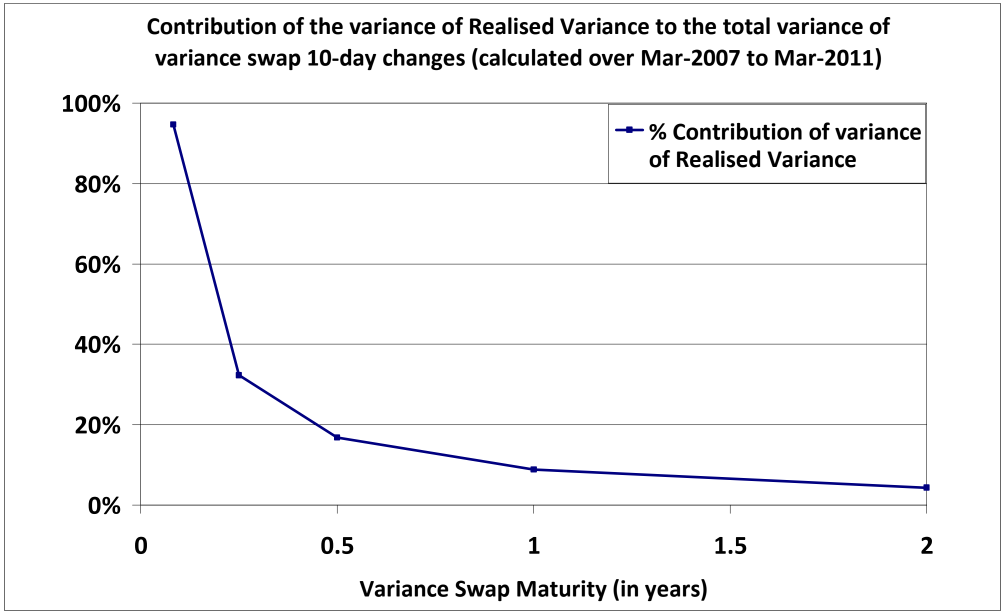
Realised variance over a short time period

$$RV^2(t, t+\Delta t)\Delta t \approx \left(\frac{\Delta S}{S(t)}\right)^2$$

- This is generally negligible for diffusion movements over small Δt, but it will be significant for large **jumps** in the underlying spot
- Realised variance will dominate expected variance moves in case of large jumps and this will drive the tail risk for variance swap

- The contribution of the realised variance is particularly important for short term variance swaps

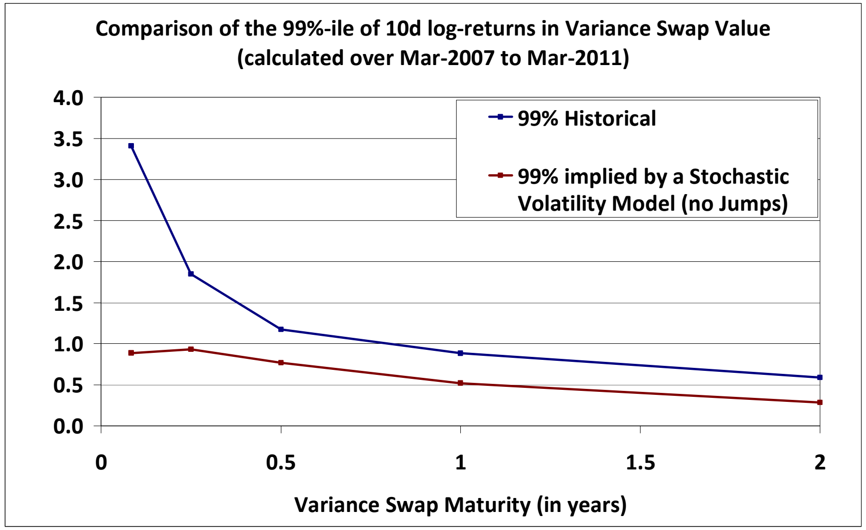
$$\%\ \text{Contribution} = \frac{VAR\left(\frac{\Delta t}{T - t_0} RV^2\right)}{VAR(\Delta VS_t)}$$



Dynamics of Realised vs Forward : Volatility

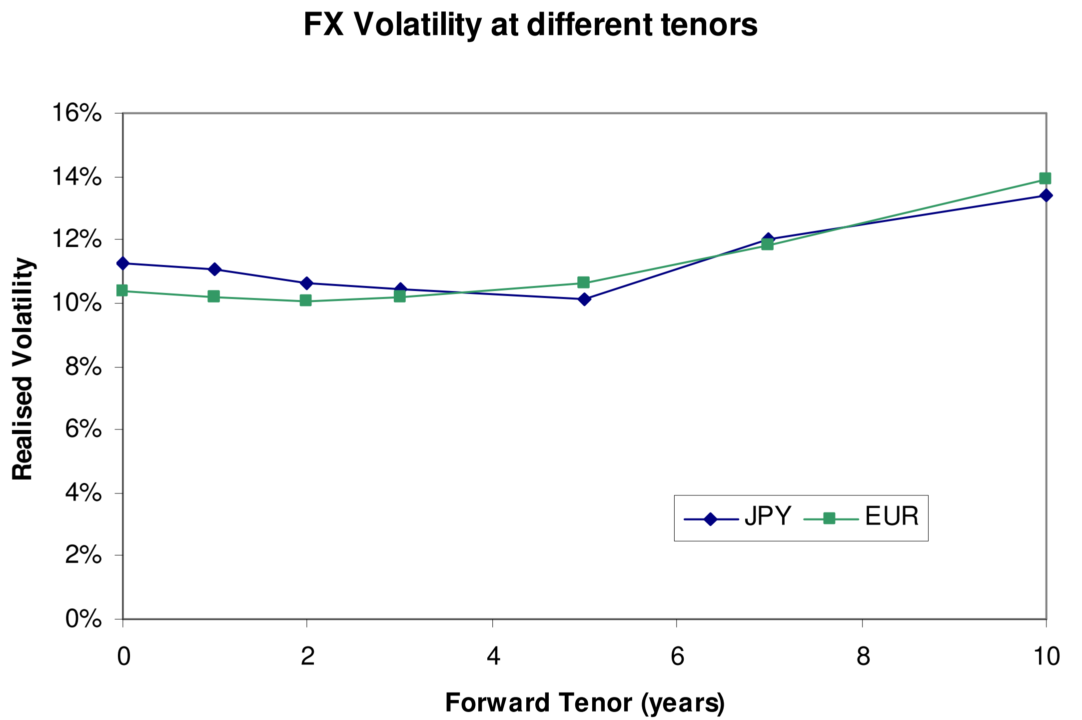
Example : Risk Management of Variance Swaps (cont'd)

- For risk management we want to capture the future realised PV moves at high confidence level
- Using stochastic volatility model (without jumps) and assuming that the realised variance can be derived from the same process as the implied variance leads to a big under-estimation of 99% 10-day moves
- We need separate (and different) processes for the instantaneous realised and implied volatilities
- Empirically we also find that over short time scales the correlation of realised volatility and implied is low.



Dynamics of Realised vs Forward : FX Rates

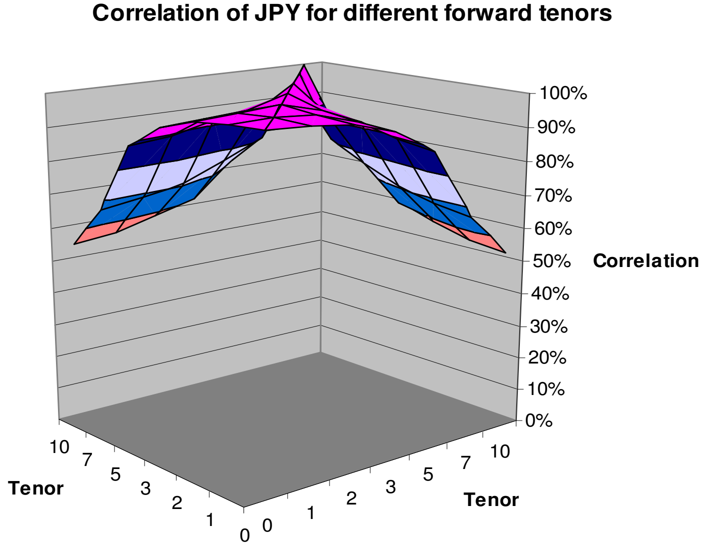
Volatility of Spot vs. Forward

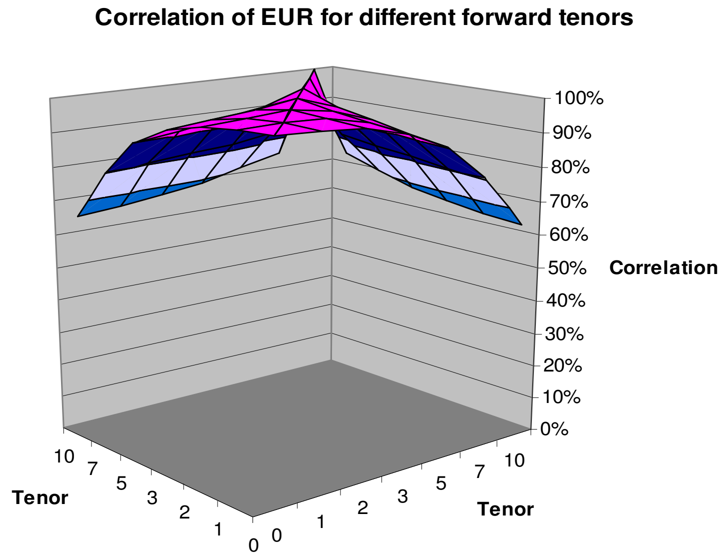


- The volatilities of EUR and JPY exhibit a slightly higher volatility for longer tenors but the function appears smooth
- This points to separate processes not being needed but rather the forward being constructed from the spot process with stochastic rate differential

Dynamics of Realised vs Forward : FX Rates

Correlation of Spot and Forward





	Spot	1YFwd	2YFwd	3YFwd	5YFwd	7YFwd	10YFwd
Spot	100%	100%	98%	95%	88%	67%	54%
1YFwd	100%	100%	99%	96%	89%	68%	55%
2YFwd	98%	99%	100%	97%	92%	71%	58%
3YFwd	95%	96%	97%	100%	93%	76%	61%
5YFwd	88%	89%	92%	93%	100%	81%	72%
7YFwd	67%	68%	71%	76%	81%	100%	81%
10YFwd	54%	55%	58%	61%	72%	81%	100%

JPY

	Spot	1YFwd	2YFwd	3YFwd	5YFwd	7YFwd	10YFwd
Spot	100%	100%	99%	96%	90%	79%	64%
1YFwd	100%	100%	99%	97%	92%	80%	65%
2YFwd	99%	99%	100%	98%	94%	82%	67%
3YFwd	96%	97%	98%	100%	96%	83%	68%
5YFwd	90%	92%	94%	96%	100%	84%	71%
7YFwd	79%	80%	82%	83%	84%	100%	75%
10YFwd	64%	65%	67%	68%	71%	75%	100%

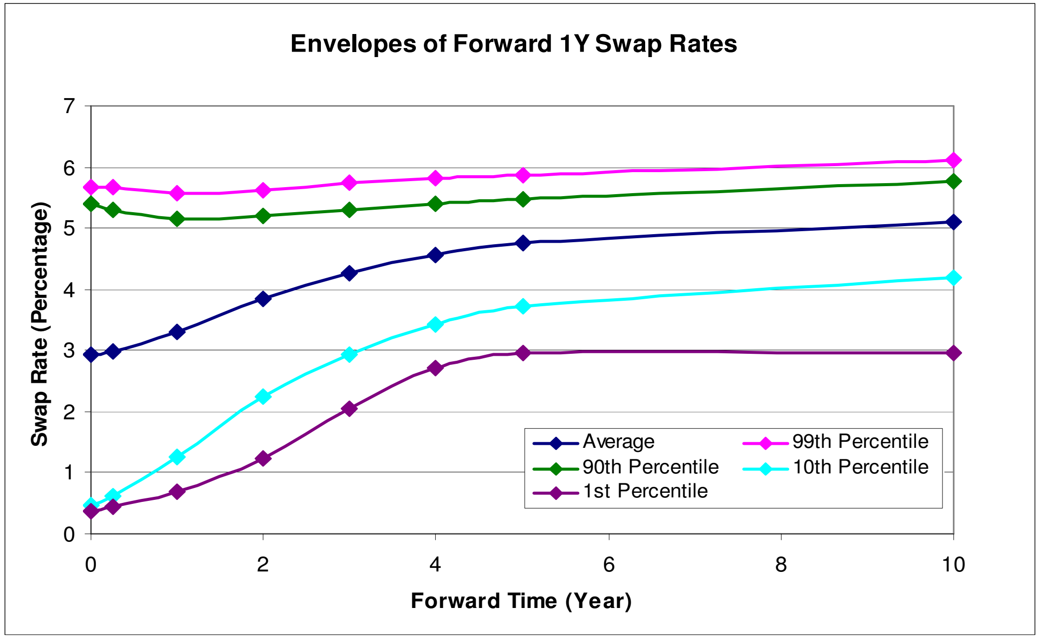
EUR

- Again no disconnects – looks consistent with stochastic rates

Dynamics of Realised vs Forward : Interest Rates

- Spot vs forward - no surprises:
  - Forward “premium” (almost monotonic increase)
  - The percentile distribution is narrower at longer forward time horizons

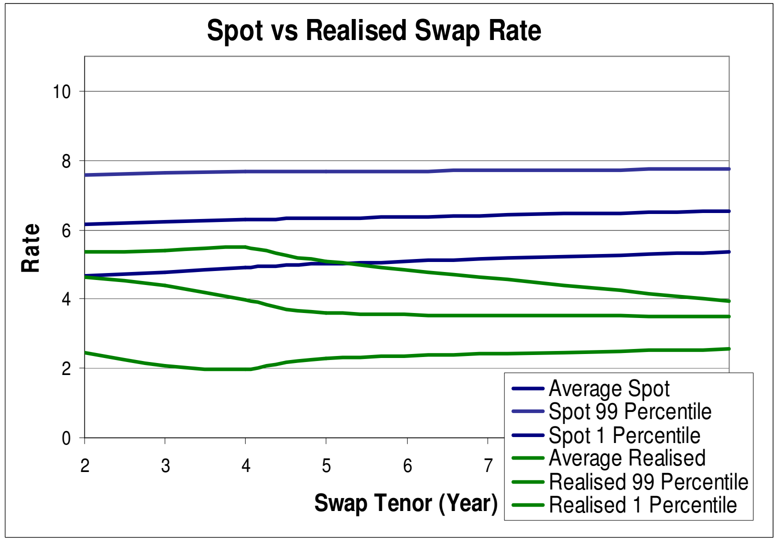
- Dynamics can be captured by a standard pricing process, no need for separate realised process

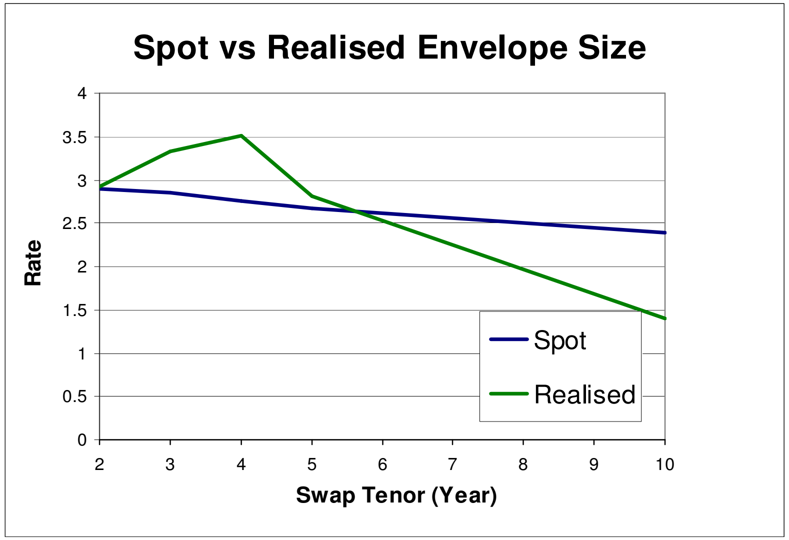


Spot vs forward - forward 1 year swap rates (USD) over 4 years (03/2007 - 03/2011)

Dynamics of Realised vs Forward : Interest Rates

- Forward rate vs. realised rate - different dynamics?

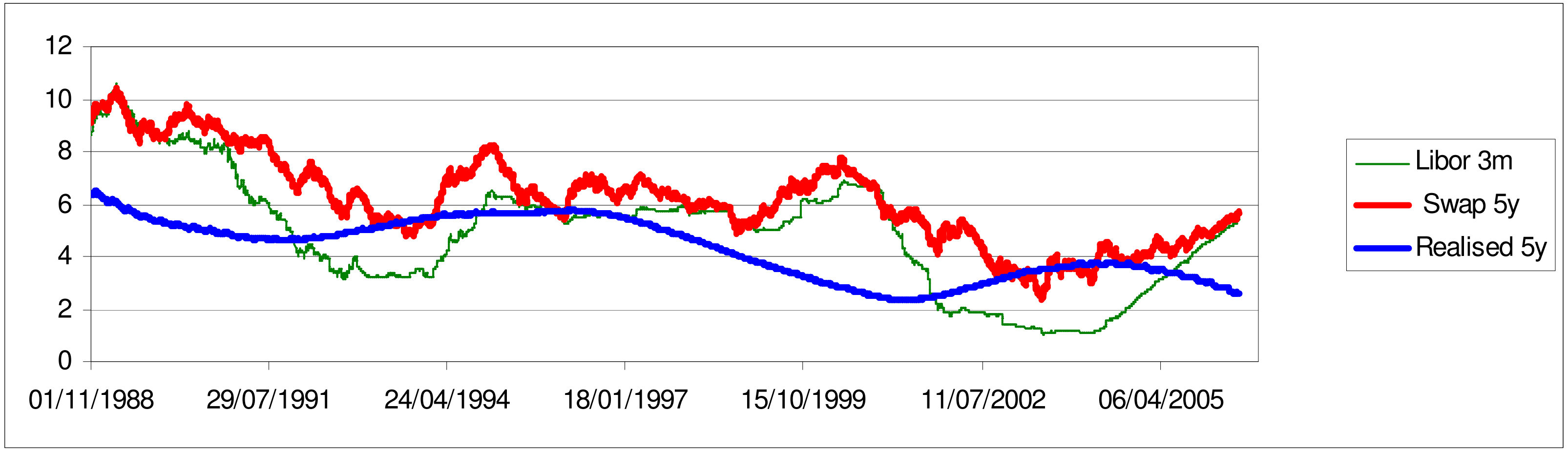


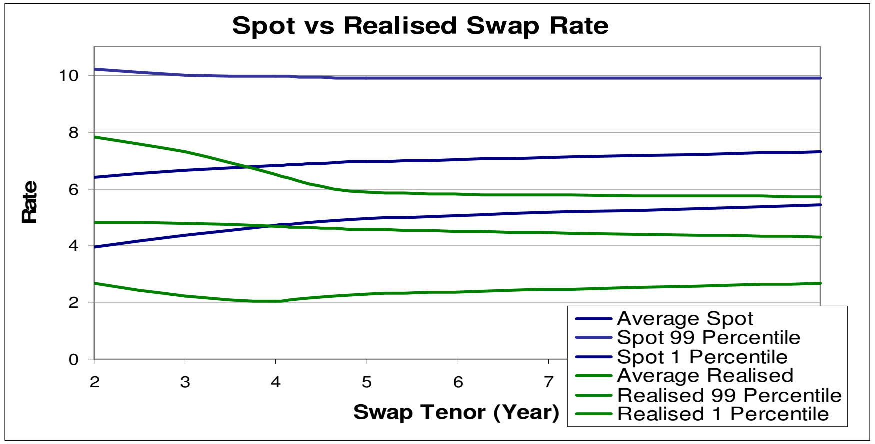


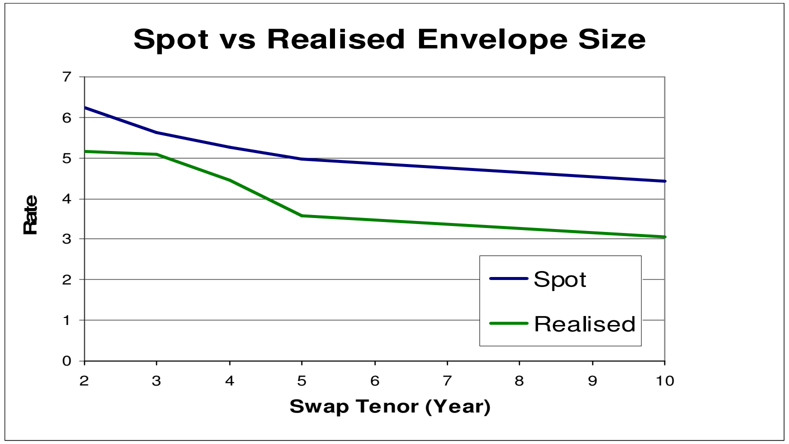
Swap tenors 1y, 2y, 3y, 4y, 5y and 10y; 4 year window. The realised swap rates are calculated from the 3m USD LIBOR rates.

- Forward rate predictor (swap rate) vs. realised rates - no clear pattern over 4 years
- Through the cycle will be different?

Dynamics of Realised vs Forward : Interest Rates







- Forward rate predictor (swap rate) vs. realised rates - through the cycle (13 years). Some evidence of different dynamics – market predictor premium exists.
- Unlike inflation, if premium not captured - minimal effect on majority of portfolios

Agenda

- Overview
- Variance Predictors : Implied and Historical data
- Expectation Predictors : Forward Prices and Alternatives
- Limitations of Stochastic Processes : A Few Interesting Examples…
- Dynamics of Realised and Forward Asset Values
- **Some Final Remarks!**

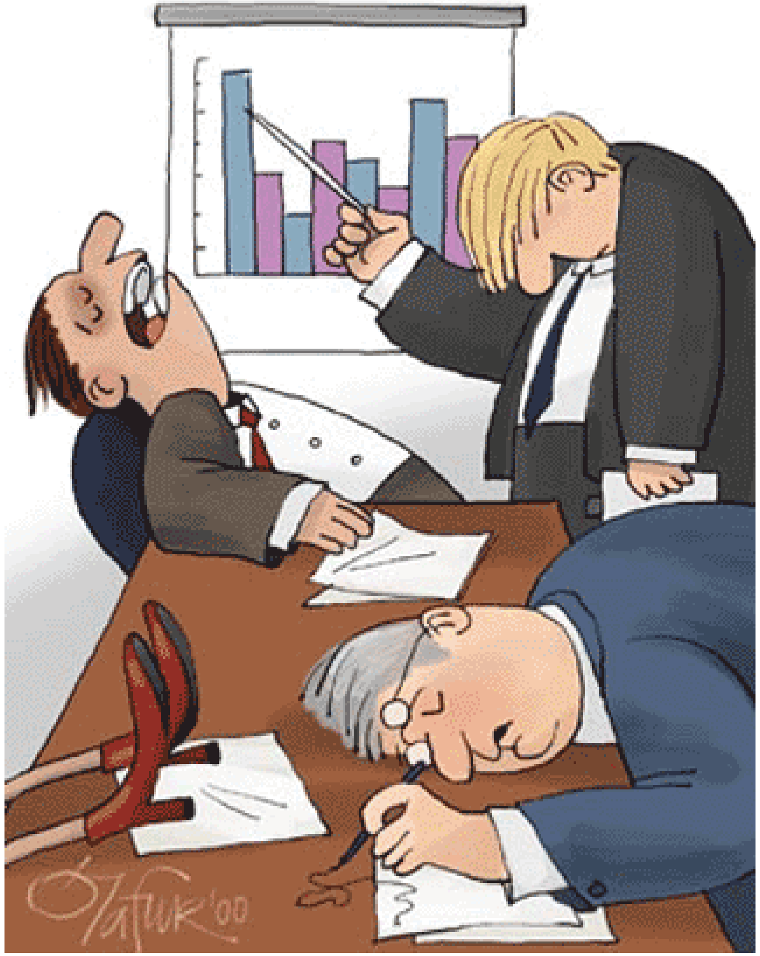
Some Final Remarks

Both real world and “risk free” worlds exist and are interconnected in a variety of ways

- There is no single winning strategy
- There is no “right” approach per asset class
- The choice should be driven by what is required to achieve and at what cost
  - How the measure is defined by the institution?
  - Whether and how it is hedged? (think CVA!)
  - What is the horizon to which it is calculated and managed?
  - Should the complexity of the model be increased to reflect rare positions?
  - How highly do we value transparency in the model?

Risk manager’s advice: listen to both worlds, trust neither!

Questions?



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