Stochastic Volatility vs. Jumps
Role in Equity-Credit Modelling
for Risk Management

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Agenda

- Volatility as Driver of Equity and Credit
  - Statistical evidence: levels and returns
  - Focus on volatility skew
- Historical Measures of Volatility Skew
  - Stochastic volatility: correlation between returns of equity and their variance
  - Jump-diffusion: jumps in equity returns
  - Co-existence of two phenomena
- Theoretical Explanations for Links between Volatility and Equity/Credit
  - Structural considerations (Merton)
  - Leverage effect
  - Jump-to-ruin
- Analysis of Effects
  - Levels and returns vs. jumps and leverage
  - Implied and historical measures
  - Cross-asset effects
- Suggestions and conclusions
- Working paper forthcoming on SSRN

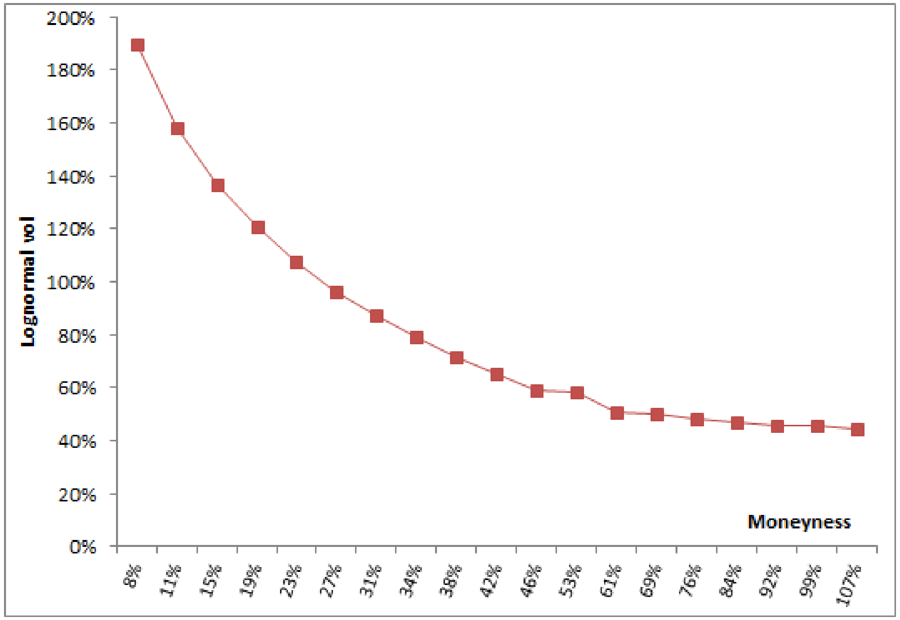
What links equity volatility to equity and credit?

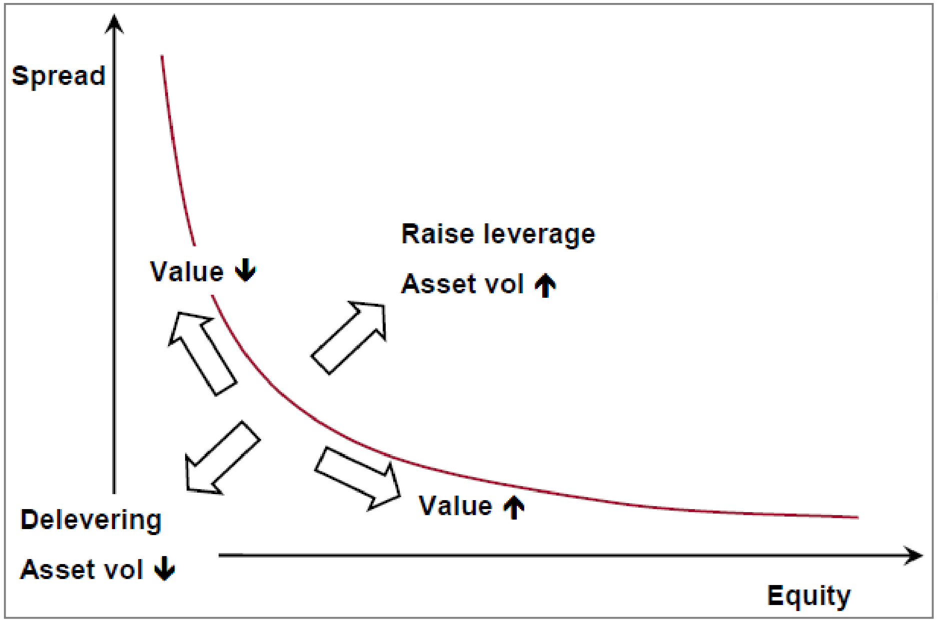
- We are interested in understanding the fundamental links between equity, credit and volatility [of equity returns]
  - See our 2014 conference presentations and recent papers on SSRN
- We start by focussing on an important element of volatility: the skew
- We have introduced a way to separate two mechanisms driving volatility skew in historical space: “Correlation” and “Jumps”
  - Inspired by two classical volatility models:
    - Stochastic volatility (Heston, 1993): variance-to-spot return correlation
    - Jump-diffusion (Merton, 1976): size and intensity of jumps
  - By analogy in historical space, introduce time-series estimates of
    - Correlation between equity returns and returns on their variance (“correlation”)
    - Average frequency times size of jumps in equity returns (“jumps”)
- We want to understand which mechanism works and how they interact
- Lipton, 2002*: a broad look at local and stochastic volatility, jump-diffusion and universal models for FX, for pricing and hedging/risk management
  - Conclusion: “*only models that take into account local, jump and stochastic features of the volatility dynamics and mix them in the right proportion are adequate*”
- We would like to test the “Lipton hypothesis” empirically

* Lipton, A., 2002. The Vol Smile Problem. Risk, February, Issue 2, pp. 61-65.

Equity volatility: the story in two graphs

- Top right: typical equity volatility skew shape ("smirk")
  - Shown: RDN 6m implied volatilities from CBOE quotes Aug 2013
- We have a tool to understand what is going on in the low-strike region





- Bottom left: the equity-credit "hockey stick" curve
  - As shown, e.g., in Richard Martin's presentation (QCE 2009)
- "Equity down, Spreads up" – but what is the role of volatility?

Volatility as a common driver for equity and credit

- *See Chorniy & Greenberg, 2015 (http://ssrn.com/abstract=2708143) for a detailed literature review and all references used in this presentation*
- Structural model: Merton (1974) asset return construction
  - Equity = Call, Debt = Put on asset value of the firm
- Volatility as a risk indicator can affect market prices of equity and debt
- Campbell & Taksler (2003): booming stock market in 1990s accompanied by rising corporate bond yields – counterintuitive?
  - Optimism of equity investors not shared by bond investors
  - Volatility may be the key: more upside for shareholders, more risk for bondholders
- Share prices and volatility of returns
  - "Leverage effect": price growth is less volatile than price drops
  - Historical volatility commonly used as a predictor of future returns distribution
- Cremers et al. (2008) : implied volatility affects credit spreads
  - Both ATM and OTM/skew explain a significant part of CDS spread levels
- Carr & Wu (2009, 2011): economic similarity between CDS and deep OTM equity puts ("Jump to ruin")

Risk factors and dependence: what to model and how

- Short-term co-movement: returns
  - Returns are best for describing/predicting underlying moves over short horizons
  - Correlated diffusions or common jumps to model joint behaviour
- Long-term predictions: levels
  - Trends matter much more for long horizons: diffusive moves average out (also $O(\sqrt{\Delta t}) \ll O(\Delta t)$), effect of jumps is short-lived
  - Classical example: long-term mean of an Ornstein-Uhlenbeck process
  - Levels can be used to enforce "pathwise" dependence (e.g., in scenarios with low share prices, spreads should be high)
- Co-movements
  - Short-term: correlation of returns; long-term: dependence between levels
  - Common driver: functional/structural dependence behind two risk factors
  - Regime shifts, intertemporal dependence
- Market-implied vs. Historical measures
  - Historical data reflect "real-world" information (free of market risk premia)
  - Implied embed market's views on probability of certain events (e.g., equity crash or issuer default)

Statistical relationships we can measure (I)

- Linear regression
  - R^2 indicates strength of relationship
  - Slope indicates direction of dependence (e.g., higher vol ➔ wider spreads and lower equity returns)
- Various combinations to analyse
  - CDS and equity levels and returns…
  - … regressed on implied and historical volatility and skew levels and returns

- Question 1: Historical volatility and skew measures
  - ATM "analogue": standard deviation of equity returns, commonly used as "proxy"
  - Volatility skew – stochastic volatility (Heston, 1993) paradigm: historical correlation between equity returns and returns on their variance
  - Volatility skew – jump-diffusion (Merton, 1976) paradigm: historical estimate of average size times intensity of downward jumps in equity returns

➢ Answer: use both, compare effects and "domains of influence"

Statistical relationships we can measure (II)

- Question 2: Measure of implied volatility skew
  - Deep OTM volatility: potentially richer information about extreme moves (jump-to-ruin), but low liquidity + extrapolation introduces noise
  - OTM – ATM (“smirk” only) or OTM – ITM (the “skew in the smile”)
- ➢ Answer: use both (results show little difference/benefit of extrapolated DOTM)

- Question 3: Measuring levels
  - CDS spreads: meaningful and universal indicator of default risk
  - Equity prices: not comparable due to “size effect”, also geographical/FX dispersion
  - Statistical point: time series must be stationary to be used in OLS regressions
  - Defining “equity level” as share price normalised by relevant index quote substantially reduces undesired effects and improves comparability
- ➢ Answer: CDS spreads and “equity levels” (normalised prices) are used

- Question 4: Relationship between implied and historical skew measures
  - Distinction between “correlation” and “jump” effects is not as clean in implied skew
  - Actual regressions on OTM-ITM vs. OTM-ATM do not “separate”
- ➢ Answer: regress implied volatility skew on historical “proxies” to see which effect dominates

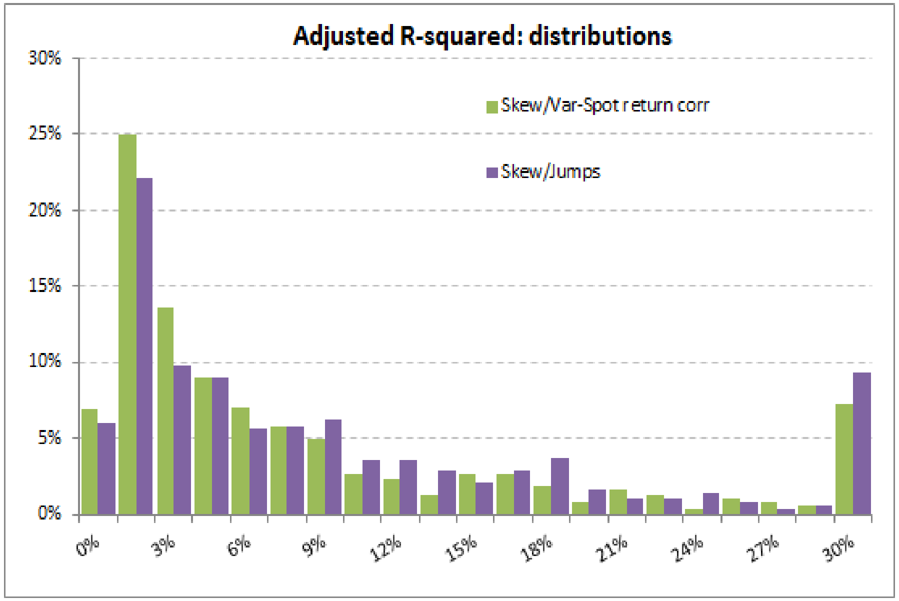
Measurement: the boring details

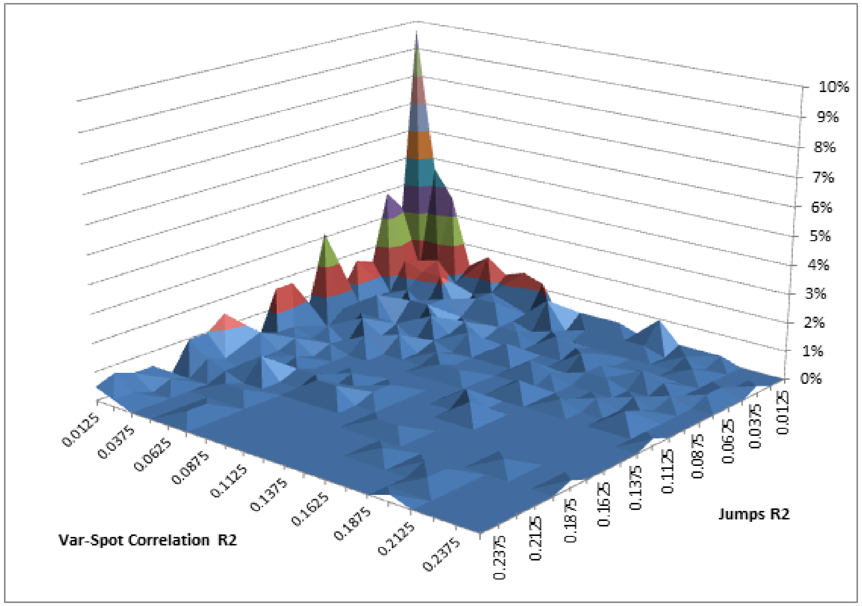
- Universe:
  - Ca. 500 names from major international equity indices with liquid CDS
  - Time series over 7 years: from September 2006 to August 2013
  - Liquid subset: ca. 160 names with well-populated CDS, equity and implied vol marks since 2007
  - Equity levels: ca. 200 names for normalised prices
  - Source: BNP Paribas

- Implied volatilities:
  - 6m option implied ATM vols
  - Deep OTM 6m put vols (30% and extrapolated to 10% strike)
  - “Smirk” (deep OTM) as (DOTM – ATM) / (10% - 100%) < 0 for equity
  - “Skew” (no “smile”) as (OTM – ITM) / (30% - 130%) < 0 for equity
- Historical volatilities:
  - Standard deviation of 10-day returns, estimated over 6 months and annualised
  - Correlation of equity returns with variance returns measured over 6m window (“correlation”)
  - Time-averaged jump measures over 6m windows (“jumps”)

Historical skew information: correlation, jumps or both? (I)

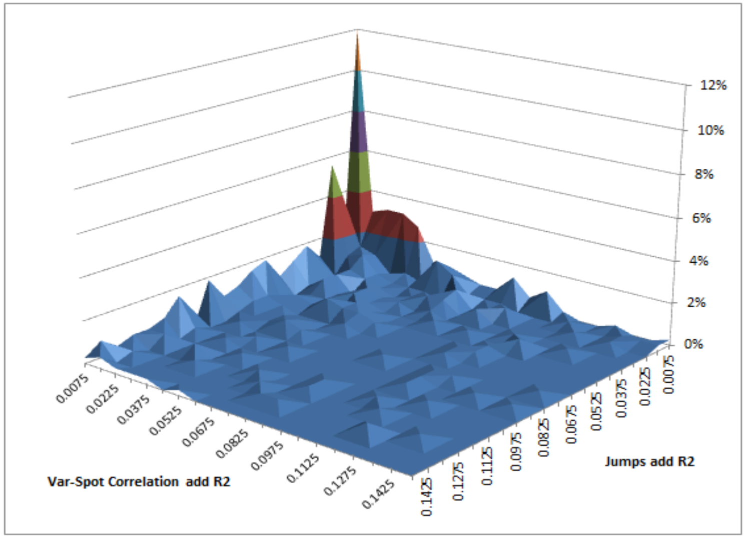
- Regress implied on historical volatilities, compare R^2's
  - ATM on historical vol levels dependence is high (65% median R^2 – not shown)
  - Regress OTM skew on the two historical "proxies": "correlation" and "jumps"
  - Much weaker dependence overall, but jumps give a larger number of high values: median R^2 at 4.7% (mean at 8.3%), vs. 3.2% (6.6%) for the "correlation" proxy
- 3D plot of the *increase* in R^2 distribution when skew variable is added
  - Clustering is around the axes, rather than in the middle, which implies that one of the two effects usually dominates
  - Concentration higher at the "Jumps" axis, so jumps look more significant



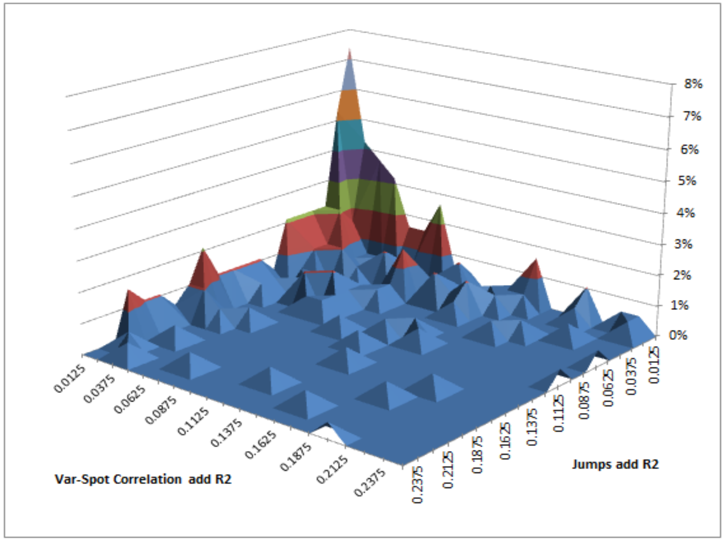


Historical skew information: correlation, jumps or both? (II)

- Compare CDS and equity level regressions on two historical skew "proxies"
  - Similar 3D plots of the *increase* in R^2's when skew variable is added
  - Compare how "correlation" (~stochastic volatility models) and "jumps" (~jump-diffusion models) historical proxies for volatility skew interact for the same names
- Clustering near the axes still present, for both CDS and equity, as above for skew, reinforcing the suggestion of "mutually exclusive" effects
- More clustering around the "jump" axis, consistent with assessment above
  - Equity level plot (right) in particular suggests that jump-diffusion language is better than stochastic volatility in historical measure



CDS levels



Equity levels

CDS and equity regressions on volatility: results

- Regression R^2 for CDS and equity on all volatility measures: all names…

Regression R^2 Median		**Implied volatility**			**Historical volatility**		
		ATM vol only	ATM + skew DOTM-ATM	ATM + skew OTM-ITM	Hist vol only	Hist vol + Correlation	Hist vol + Jumps
Equity	Returns	**30%**	**30.5%**	**31.5%**	2.5%	2.5%	4.5%
	Levels	10%	**24.5%**	**28%**	9.5%	**18%**	**19%**
CDS	Returns	7.5%	8%	8.5%	2%	2%	2.5%
	Levels	**31.5%**	**46.5%**	**52%**	**23%**	**32.5%**	**34%**

- … and liquid subset of names (except for equity levels)

Regression R^2 Median		**Implied volatility**			**Historical volatility**		
		ATM vol only	ATM + skew DOTM-ATM	ATM + skew OTM-ITM	Hist vol only	Hist vol + Correlation	Hist vol + Jumps
Equity	Returns	**39.5%**	**40%**	**42%**	1.5%	2%	3%
	Levels	*10%*	*24.5%*	*28%*	*9.5%*	*18%*	*19%*
CDS	Returns	**19.5%**	**20.5%**	**20%**	2%	2%	3%
	Levels	**45.5%**	**54%**	**52.5%**	**40%**	**46.5%**	**52%**

CDS and equity regressions on volatility: comments

- Relationship is stronger for implied than for historical volatility measures
- Volatility skew measures (of any flavour) are important for levels, but not for returns
- Dependence is stronger for levels than for returns, *except* for equity on implied volatilities, where dependence of returns is stronger
  - Recall time scale for dependence: returns = short-term, levels = long-term
- Liquid subset (second table) emphasizes the same dependence pattern
  - Significant improvement for CDS returns on implied vol returns

- Direction of dependence: higher volatility ➔ wider CDS, lower equity
  - This is true for both levels and returns (verified by looking at regression slopes)

- Jumps as indicator of historical information for volatility skew play a part in explaining levels for both CDS and equity
  - This is consistent with 3D plots presented before
- No significant difference between two “flavours” of implied volatility skew
  - No visible “bias” of OTM-ITM towards returns or DOTM-ATM towards jumps

Volatility skew drivers: recap

- We introduced two historical proxy variables for volatility skew:
  - Correlation between equity returns and returns on their variance (à la stochastic vol)
  - Average size times intensity of jumps in equity returns (à la jump-diffusion)
- Both have explanatory power for implied volatility skew, as well as for CDS spreads (levels)
  - Results indicate a stronger influence of the jump variable
- We defined "equity levels" as index-normalised share prices
  - Removes "size effect" and improves stationarity of time series
  - Recovers similar dependence as for CDS spreads: skew adds explanatory power
  - Stronger influence of the jump variable compared to correlation variable, as for CDS
- For both equity and CDS spread levels, jump and correlation variables are not independent effects
  - Either one or the other is likely to drive volatility skew of any given name

- Thus judging by effects on the skew, the "Lipton hypothesis" can be challenged:
  - One out of stochastic volatility and jump-diffusion could be sufficient
  - Jump-diffusion is a favourable contender
- We look at wider mechanisms and predictions in more detail next

Beyond skew: stochastic volatility or jumps in our model?

Everything should be made as simple as possible, but not simpler. – A. Einstein

- A good [risk] model should reflect reality as much as possible
  - Material risk factors should be included
  - Dependencies which represent existing fundamental links will be more stable
  - Model type/approach can also be tailored to observed risk factor interactions
- Volatility models: stochastic volatility vs. jump-diffusion
  - Recall Lipton, 2002, for FX volatility models: mixture of all features required
  - We ask the question again in the context of equity-credit-volatility risk models
  - Simple regressions of skew indicate that the two features are orthogonal
  - Combine this information with theoretical predictions in a wider volatility model context, to refute or reinforce the preliminary finding
- Question: how do stochastic volatility and jump features interact in general?
  - Are jump and stochastic volatility effects significant for the same names?
  - Do the effects propagate across asset classes (equity, credit spreads) and dependency types (short-term, long-term)?
- Practical considerations: what model to invest in with limited resources?
  - Is a rich “hybrid” model a must?
  - Is any one feature sufficient and not too simplistic?

Volatility, Equity and Credit: theoretical models

- Three main interpretations of dependence between CDS spreads and equities, and volatilities
- “Pure” Merton, 1974: link via asset (i.e., firm value) volatility, which is a measure of investment risk
  - Equity = long call, Debt = short put on asset value
  - Equity volatility is a function of asset volatility, or is asset “vol of vol”
- “Leverage”: general term for equity volatility inversely related to equity returns
  - Financial leverage: debt-to-equity ratio of the firm, so drop in stock price increases leverage ratio
  - Share price drop $\Rightarrow$ higher leverage $\Rightarrow$ riskier stock $\Rightarrow$ more volatile stock
  - Alternatively, volatility is a measure of [investment] risk
  - Higher risk requires higher return, so stock price needs to drop first (volatility increase should be persistent)
  - CDS also reflects the riskiness of investment (spread over “risk-free” investment)
- Equity volatility embeds a measure of probability of jump-to-default
  - Excess return of equity over risk-free rate compensates for possible default
  - CDS reflect market view of the default probability

Volatility, Equity and Credit: predictions

- Merton and firm value volatility
  - "Time value" of Merton options: changes convexity of equity-credit "hockey stick"
    - $\Rightarrow$ Volatility increase means both equity and CDS spread increases
  - "Vol of asset vol": stochastic volatility model predictions
    - $\Rightarrow$ Fatter tails make options more expensive, so both equity and CDS spreads increase (but less)
    - Equity investors like the risk (long vol), bond investors do not (short vol)
  - So for Merton, **Vol *up*** $\Rightarrow$ **CDS *up*, Equity *up***
  - Stronger *longer term impacts* expected if "time value" effect for the vol link; but always weaker than Equity-Credit predictions (below)

- Leverage: more immediate dependence, expected to dominate in the short term
  - Expect CDS, equity and volatility *returns* to be correlated
  - "Hockey-stick": when "Merton" reduces convexity the graph, "Leverage" moves to another curve
  - So for leverage, **Vol *up*** $\Rightarrow$ **CDS *up*, Equity *down***

- Link via default, or jump-to-ruin information:
  - CDS, equity and volatility *levels* should be related
  - **Vol *up*** $\Rightarrow$ **CDS *up*, Equity *down***

Volatility, Equity and Credit: results vs. predictions

- Empirically measured direction of dependence remains consistent for implied and historical volatilities, for levels as well as for returns
  - Larger volatility is associated with wider CDS spreads, and likewise for returns
  - Larger volatility is associated with lower equity prices and returns
- ➢ This is consistent with both “leverage” and “jump to ruin” interpretations

- Empirically, volatility skew is relevant for levels but not for returns
  - We expect ‘*leverage*’ to drive *short-term* behaviour (*returns*), while ‘*jump to ruin*’ information is embedded in values and is realised over *longer term* (*levels*)
  - Historical vols contribute to levels only, while implied vols contribute to returns
  - Also OTM skew (cf. deep OTM puts) is indicative of stock crash/default risk
  - Jump are dominant “driver” of skew
- ➢ This is consistent with dependencies between returns being driven by leverage, and dependencies between levels being driven by expected and realised jumps
- Merton explanation (*“vol up ➔ CDS, equity up”*) does not work
  - “Vol up ➔ Equity up” dependency could re-appear when time-dependent correlations or intertemporal dependence is considered

Returns vs. Levels and Leverage vs. Jumps

- Evidence of Jumps affecting levels (long-term)
  - Jumps are associated with CDS (jump to ruin probability) and OTM vol or skew
    - Observed jumps in the past fuel expectation of even more drastic jumps in the future
  - Strongest relationships by median R^2: CDS on volatility *levels*
  - Relationship persists: both for implied and historical (i.e., over longer term) vols
  - Even normalised equity (*levels*) reach significant R^2 against OTM vol *levels*
  - OTM skew regresses somewhat better on the historical jump measure
- Evidence of Leverage affecting returns (short-term)
  - Leverage manifests itself in variance-to-equity return correlation (negative skew)
  - Strongest relationships by median R^2: equity on implied vol *returns* (Leverage)
  - Returns R^2 unaffected by skew (~Jumps), so they must be driven by Leverage

- However separation is not clean, at best approximate:
  - CDS on implied volatility *returns* regression significant for liquid names
  - Historical measure of ‘Leverage’ skew (variance-to-equity return correlation) adds explanatory power to CDS level regressions as well
  - Equity “levels” (normalised prices) also regress well on ‘correlation’ historical skew proxy (and on OTM-ITM skew)
- Two “flavours” co-exist – but are they complementary or mutually exclusive?

Testing co-existence of Leverage and Jumps

- Assuming that one effect dominates levels and the other, returns
  - Mutually exclusive: low 'levels R^2' for names with high 'returns R^2' (and vice versa)
  - Mutually supporting: R^2's for levels and returns tending to be low or high together
  - Co-existent, no relationship: "random" pattern across level and return R^2's
- However we need to test the dominance hypothesis
  - If the same effect (Jumps) dominates levels, then all level R^2's, across asset classes and volatility types, will tend to be high or low "together" (i.e., for the same names)
  - Similarly for returns, with an added twist: expect alignment of R^2's *within* implied and historical volatility regressions, but not *across* (relationship not persistent)
- The above can be checked by looking at scatter plots of R^2's and sign of the correlation between the corresponding vectors:
  - Mutually exclusive (negative correlation): scatter plot will have clusters along axes
  - Mutually supporting/same driver (high positive correlation): scatter plot arranges along a [straight] line
  - No relationship (low/near-zero correlation): uniformly distributed scatter plot
- This relates to the "Lipton hypothesis" of what is required of a model
  - Mutually exclusive means only one effect is at play – "universal model" not required
  - Mutually supporting or none mean both are important – "universal model" is needed

NB: look at regressions on ATM and historical vols only (to remove any skew bias)

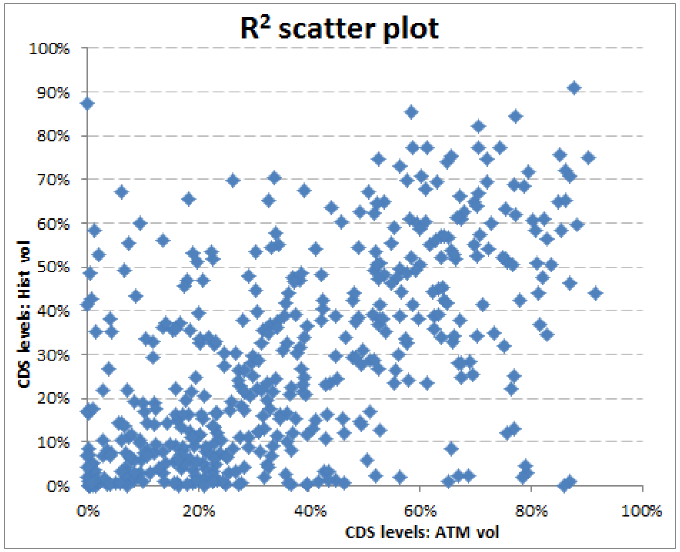
Level and return regression drivers: mixed results

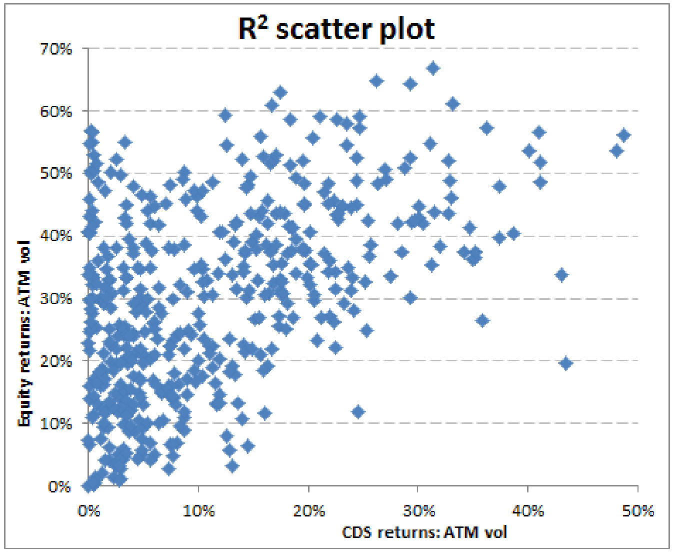
Correlations between R^2 vectors			Levels				Returns			
			CDS		Equity		CDS		Equity	
			ATM	Hist	ATM	Hist	ATM	Hist	ATM	Hist
Levels	CDS	ATM	100.00%	57.82%	9.51%	3.19%	52.51%	14.06%	23.75%	4.64%
		Hist	57.82%	100.00%	9.16%	12.05%	25.67%	27.64%	16.56%	11.69%
	Equity	ATM	9.51%	9.16%	100.00%	68.66%	-1.90%	-3.93%	-1.19%	-10.19%
		Hist	3.19%	12.05%	68.66%	100.00%	-7.54%	5.70%	1.67%	-9.77%
Returns	CDS	ATM	52.51%	25.67%	-1.90%	-7.54%	100.00%	16.07%	46.29%	4.17%
		Hist	14.06%	27.64%	-3.93%	5.70%	16.07%	100.00%	5.41%	38.54%
	Equity	ATM	23.75%	16.56%	-1.19%	1.67%	46.29%	5.41%	100.00%	26.88%
		Hist	4.64%	11.69%	-10.19%	-9.77%	4.17%	38.54%	26.88%	100.00%

- Dominance hypothesis:
  - Levels dependence should be persistent (implied and historical vols) – TRUE
  - Levels dependence should propagate across asset classes – FALSE
  - Returns dependence aligned within volatility type: TRUE across asset classes
- Assuming ‘jumps’ explain levels and ‘leverage’ explains returns
  - CDS show “mutually supportive” pattern, Equity show “mutually exclusive” pattern

- Very few cases of interdependence, one “mutually exclusive” (weak)
- Therefore we do need both effects in the model: “Lipton hypothesis” stands!

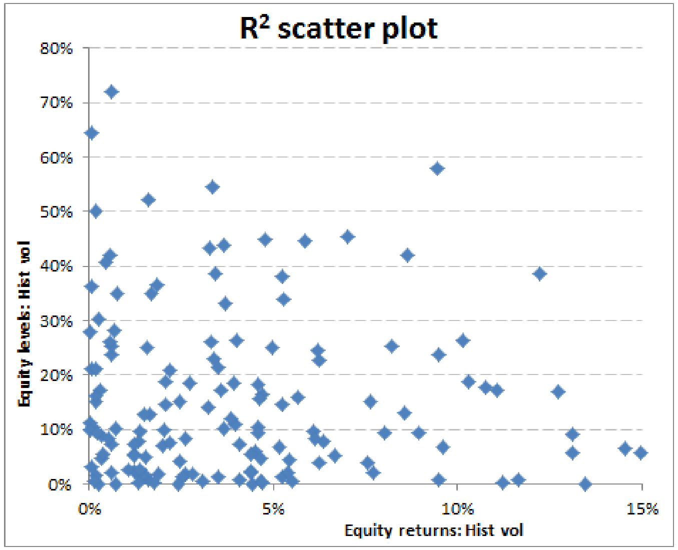
Level and return regression patterns: examples



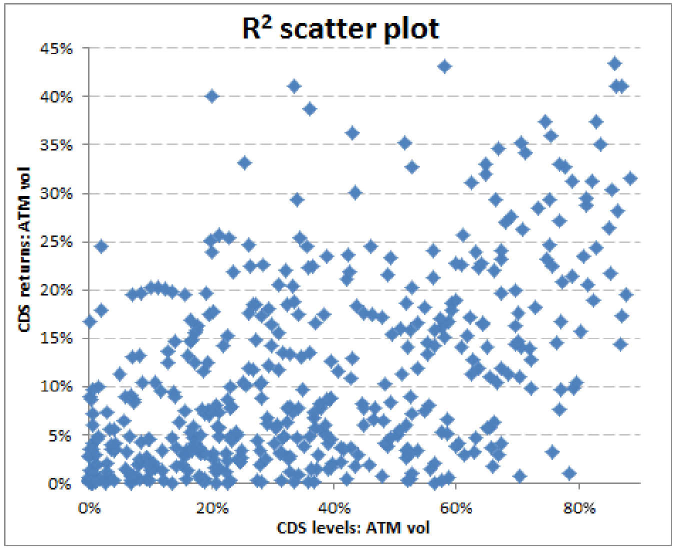


Alignment:

- CDS levels (left): implied vs. hist
- CDS vs. equity (right): on implied



- Mutually exclusive (left): equity returns vs. levels
- Mutually supporting (right): CDS returns vs. levels on implied vol



Conclusions and further research

- We introduced a way of measuring historically the significance of contributions from jumps and stochastic volatility effects to volatility skew
  - Correlations between equity returns and returns on their variance (~leverage)
  - Average downward jump size times intensity (~jumps)
  - Both play a role, jumps somewhat more influential for levels of equity, credit and implied volatility skew
  - Jumps and stochastic volatility tend to be mutually exclusive “drivers” of vol skew

- Analysis of fundamental relationships in a broader volatility context:
  - Stochastic volatility and leverage; jump-diffusion and jump-to-ruin expectations…
  - Levels as better links to jumps and returns, to leverage (as an approximation)…
  - Dependence between returns and levels, within and across asset classes…
- … shows little interplay between the two effects, so need to be captured
  - Lipton’s conclusion for FX pricing models is supported
- Extending the analysis
  - Other asset classes (FX is a good candidate, specifically to understand skew)
  - Some evidence observed:
    - Regime shift: possibly different patterns for extreme returns
    - Longer-term intertemporal dependencies limit regression as a tool

The views expressed by authors in this presentation are their own and do not necessarily reflect the views of BNP Paribas.